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The chiral model of Sakai-Sugimoto at finite baryon density

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Abstract: In the context of holographic QCD we analyze Sakai-Sugimoto's chiral model at finite baryon density and zero temperature. The baryon number density is introduced through compact D4 wrapping S^4 at the tip of D8-$\overline{\mathrm{D8}}$. Each baryon acts as a chiral point-like source distributed uniformly over $\mathbb{R}^3$, and leads a non-vanishing $\mathrm{U}(1)_V$ potential on the brane. For fixed baryon charge density n_B we analyze the energy density and pressure using the canonical formalism. The baryonic matter with point like sources is always in the spontaneously broken phase of chiral symmetry, whatever the density. The point-like nature of the sources and large N_c cause the matter to be repulsive as all baryon interactions are omega mediated. Through the induced DBI action on D8-$\overline{\mathrm{D8}}$, we study the effects of the fixed baryon charge density n_B on the pion and vector meson masses and couplings. Issues related to vector dominance in matter in the context of holographic QCD are also discussed.

Keywords: AdS-CFT Correspondence, Phenomenological Models.

Contents

1. Introduction

Dense hadronic matter is of interest to a number of fundamental problems that range from nuclear physics to astrophysics. QCD at finite baryon density is notoriously difficult: (1) the introduction of a chemical potential causes most lattice simulations to be numerically noisy owing to the sign problem; (2) the baryon-baryon interaction is strong making most effective approaches limited to subnuclear matter densities.

In the limit of a large number of colors N_c, QCD is an effective theory of solely mesons where baryons appear as chiral skyrmions. Dense matter in large N_c is a skyrmion crystal with spontaneous breaking of chiral symmetry at low density, and restored or stripped (Overhauser) chiral symmetry at high density. While some of these aspects can be studied qualitatively using large N_c motivated chiral models [1], they still lack a first principle understanding.

The AdS/CFT approach has provided a framework for discussing large N_c gauge theories at strong coupling $\lambda = g^2 N_c$ from first principles [2]. A particularly interesting AdS/CFT construction is the holographic and chiral approach proposed by Sakai and Sugimoto [3, 4] (SS model). In the limit where $N_f \ll N_c$, chiral QCD is obtained as a gravity dual to N_f D8-$\overline{\text{D8}}$ embedded into a $D4$ background in 10 dimensions where supersymmetry is broken by the Kaluza-Klein (KK) mechanism. The KK scale plays the role of the chiral scale. The SS model yields a first principle effective theory of pions, vectors, axials and baryons that is in good agreement with experiment [3 − 5]. The SS model at finite temperature has been studied in [6] and the bayrons in the context of Skyrmion and Instanton have been worked out [5, 7].

Recently we have suggested that the SS model can be used to analyze dense hadronic matter at large N_c and strong coupling λ [8]. At zero temperature, the quark/baryon chemical potential μ (or $\mu_B = m_B + N_c\mu$) is introduced as the boundary value of the $\mathrm{U}(1)_V$ brane potential A_0. The diagonalization of the vector modes in dense matter, enforces vector dominance and yields $\mathcal{A}_0 = \mu\psi_v$ throughout where ψ_v is given in [4]. While the mode decomposition of ψ_v on the brane leads to a highly oscillating $\mathcal{A}_0$, we have argued in [8] that only those modes in $\mathcal{A}_0$ below the KK scale should be retained. As a result both brane and meson properties in holographic and dense baryonic matter were discussed.

Soon after the posting of this work, several studies appeared addressing the same issue of baryonic matter in holographic QCD including also temperature. In [9] it was suggested that the $\mathcal{A}_0$ field is instead fixed by the equation of motion on the brane by varying the pertinent DBI action. In the absence of brane "charges" the authors in [9] concluded that only a constant $\mathcal{A}_0$ is a solution, with no baryonic effect at zero temperature. However, the conserved baryonic charge has to be mirrored by the "charge" of baryon vertex. The latter is obtained by considering brane wrapping of S^4 within D8-$\overline{\text{D8}}$. The wrapping number is the bulk conserved"charge" which is at the origin of a non-constant $\mathcal{A}_0$ in bulk.

This point was further developed in [10] and used to discuss the phase structure of dense and hot holographic matter, albeit in a brane set up without chiral symmetry. In [11] it was argued that the additional "charges" in bulk upset the smoothness of the DBI surface in bulk, leading to a spiky structure due to the force balancing condition.[1] As a result, the embedded branes should *always* touch the horizon even for arbitrarily small temperature and/or density, thereby altering totally the phase diagram in [10]. This latter point is physically unintuitive. Indeed, the "charge" or baryon vertex exists even at zero temperature with no need to connect to any horizon. At finite temperature through the insertion of a black hole this point is developed in details in [12].

In this paper we follow on the analysis in [10] in bulk and at zero temperature (and low temperature before deconfinement phase transition). The dual "charge" or baryon vertex is inserted at the tip of the minimally embedded D8-$\overline{\text{D8}}$ surface. This way each of D8, $\overline{\text{D8}}$ is shared equally, leading to a *chiral* baryon vertex. Since no connecting string is involved, there is no spiky structure involved here. Also, there is a one-to-one correspondence between the baryon vertex normalization and the Wess-Zumino term in the induced chiral DBI action. Thus the boundary chiral skyrmions constructed from the induced DBI action, are dual to *static* and *point-like* instantons in bulk. These conditions will be relaxed in a sequel. In many ways, this approach complements the original discussion in [8].

In section 2, we briefly review the SS model and set up the notations. In section 3, we introduce the $\mathrm{U}(1)_V$ field $\mathcal{A}_0$ in bulk and show how the baryon charge density n_B affects its minimal profile. In section 3 and 4, we construct the bulk hamiltonian and derive the energy density as a function of the identified baryon density. The energy density is found to grow about quadratically with the baryon density. In section 5, we summarize the construction of the chiral effective action for pions, vectors and axials at zero density. In section 6, we show how this chiral effective action is modified by the finite "charges" in bulk. A number of meson properties are discussed as a function of the identified baryon number. Our conclusions are in section 7. Throughout, the canonical formalism will be used.

2. SS model

In this section we summarize the D4/D8-$\overline{\text{D8}}$ set up for notation and completeness. For a thorough presentation we refer to [3] and references therein. The metric, dilaton ϕ, and the 3-form RR field C_3 in N_c D4-branes background are given by

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)\,, \tag{2.1}$$

$$e^\phi = g_s\left(\frac{U}{R}\right)^{3/4}\,, \qquad F_4 \equiv dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4\,, \qquad f(U) \equiv 1 - \frac{U_{\mathrm{KK}}^3}{U^3}\,, \tag{2.2}$$

where $x^\mu = x^{0,1,2,3}$, $\tau(\equiv x^4)$ is the compact variable on S^1. $U(\geq U_{\mathrm{KK}})$ and Ω_4 are the radial coordinate and four angle variables in the $x^{5,6,7,8,9}$ direction. $R^3 \equiv \pi g_s N_c l_s^3$, where g_s and l_s are the string coupling and length respectively. $V_4 = 8\pi^2/3$ is the volume of unit S^4 and ϵ_4 is the corresponding volume form.

[1] Notice that while the baryonic charges are space separated, they occupy the same position in the dual transverse space!

To avoid a conical singularity at $U = U_{\rm KK}$ the period of $\delta\tau$ of the compactified τ direction is set to

$$\delta\tau = \frac{4\pi}{3}\frac{R^{3/2}}{U_{\rm KK}^{1/2}} \,. \tag{2.3}$$

in terms of which we define the Kaluza-Klein mass as

$$M_{\rm KK} \equiv \frac{2\pi}{\delta\tau} = \frac{3}{2}\frac{U_{\rm KK}^{1/2}}{R^{3/2}} \,. \tag{2.4}$$

The parameters R, $U_{\rm KK}$, and g_s may be expressed in terms of $M_{\rm KK}$, $\lambda(= g_{YM}N_c)$, and l_s as

$$R^3 = \frac{1}{2}\frac{\lambda l_s^2}{M_{\rm KK}}\,, \quad U_{\rm KK} = \frac{2}{9}\lambda M_{\rm KK} l_s^2\,, \quad g_s = \frac{1}{2\pi}\frac{\lambda}{M_{\rm KK}N_c l_s} \tag{2.5}$$

Now, consider N_f probe D8-branes in the N_c D4-branes background. With $\mathrm{U}(N_f)$ gauge field A_M on the D8-branes, the effective action consists of the DBI action and the Chern-Simons action:

$$\begin{aligned} S_{\rm D8} &= S_{\rm D8}^{\rm DBI} + S_{\rm D8}^{\rm CS}\,, \\ S_{\rm D8}^{\rm DBI} &= -T_8 \int d^9x\; e^{-\phi}\; \mathrm{tr}\sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})}\,, \end{aligned} \tag{2.6}$$

$$S_{\rm D8}^{\rm CS} = \frac{1}{48\pi^3}\int_{D8} C_3 \mathrm{tr}\, F^3 \,. \tag{2.7}$$

where $T_8 = 1/((2\pi)^8 l_s^9)$, the tension of the D8-brane, $F_{MN} = \partial_M A_N - \partial_N A_M - i\,[A_M, A_N]$ $(M, N = 0, 1, \ldots, 8)$, and g_{MN} is the induced metric on D8-branes:

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu + \left[\left(\frac{U}{R}\right)^{3/2} f(U)(\tau'(U))^2 + \left(\frac{R}{U}\right)^{3/2}\frac{1}{f(U)}\right]dU^2 + \left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2\,, \tag{2.8}$$

where $\tau' = \frac{d\tau}{dU}$. The effective action of $\overline{\rm D8}$ has the same form and the total action of N_f D8-$\overline{\rm D8}$-branes has a symmetry

$$\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R = \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V \times \mathrm{U}(1)_A\,, \tag{2.9}$$

which is interpreted as a flavor chiral symmetry of massless quarks.

In the SS model baryons are skyrmions in $\mathbb{R}^3$. However, in the gravity dual they are either an effective fermion degree of freedom in 5-dimensions (bottom-up) or an instanton wrapping $D4$ (top-down) and sourcing the baryon current through the Chern-Simons term [5, 8, 13, 14]. In both cases, the baryon can be treated as a delta function source of the brane gauge field $\mathcal{A}_0$ (which is ω_0 in $\mathbb{R}^3$), which we will use explicitly.

3. Background field $\mathcal{A}_0$

Let $\mathcal{A}_0(U)$ be a $\mathrm{U}(1)_V$ valued background gauge field in bulk. Its boundary value is related to the baryon chemical potential [8 − 10, 12]. In the absence of the source, the effective action of the D8-branes (2.6) becomes

$$S_{\mathrm{D8}} = -\frac{N_f T_8 V_4}{g_s} \int d^4x\, dU U^4 \left[f\,(\tau')^2 + \left(\frac{R}{U}\right)^3 \left(f^{-1} - \left(2\pi\alpha' \mathcal{A}_0'\right)^2\right)\right]^{\frac{1}{2}}, \tag{3.1}$$

where $\mathcal{A}_0' = \frac{d\mathcal{A}_0}{dU}$ and the Chern-Simons action vanishes. The equations of motion for $\tau(U)$ and $\mathcal{A}_0(U)$ are [9]

$$\frac{d}{dU}\left[\frac{U^4 f\,\tau'}{\sqrt{f\,(\tau')^2 + \left(\frac{R}{U}\right)^3 \left(f^{-1} - (2\pi\alpha'\mathcal{A}_0')^2\right)}}\right] = 0, \tag{3.2}$$

$$\frac{d}{dU}\left[\frac{U^4 \left(\frac{R}{U}\right)^3 \mathcal{A}_0'}{\sqrt{f\,(\tau')^2 + \left(\frac{R}{U}\right)^3 \left(f^{-1} - (2\pi\alpha'\mathcal{A}_0')^2\right)}}\right] = 0. \tag{3.3}$$

In this paper we consider only the case $\tau' = 0$, Sakai-Sugimoto's original embedding [3, 4], where the D8-branes configuration in the τ coordinate is not affected by the existence of background $\mathcal{A}_0$. This corresponds to $\tau = \frac{\delta\tau}{4}$, the maximal asymptotic separation between D8 and $\overline{\mathrm{D8}}$ branes.

To compare with [3, 4] we change the variable U to z through

$$U \equiv (U_{\mathrm{KK}}^3 + U_{\mathrm{KK}} z^2)^{1/3} \tag{3.4}$$

The action (3.1) is then

$$S_{\mathrm{D8}} = -N_f \widetilde{T} \int d^4x \int_0^\infty dz\, U^2 \sqrt{1 - (2\pi\alpha')^2 \frac{9}{4}\frac{U_z}{U_{\mathrm{KK}}}(\partial_z \mathcal{A}_0)^2}\,, \tag{3.5}$$

where we used $\tau' = 0$ and $\widetilde{T} \equiv \frac{N_c M_{\mathrm{KK}}}{216\pi^5\alpha'^3}$. It is useful to define the dimensionless quantities

$$Z \equiv \frac{z}{U_{\mathrm{KK}}}, \quad K(U) \equiv 1 + Z^2 = \left(\frac{U}{U_{\mathrm{KK}}}\right)^3, \tag{3.6}$$

in terms of which the action is written as[2]

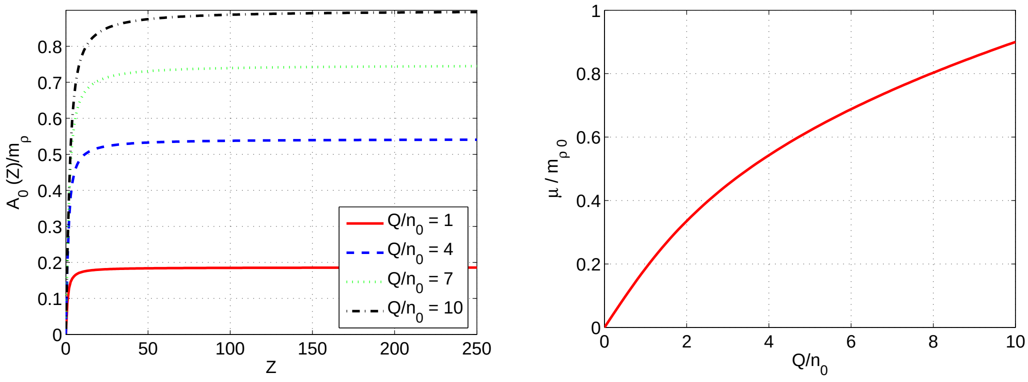
$$S_{\mathrm{D8}} = -a \int d^4x \int dZ\, K^{2/3} \sqrt{1 - bK^{1/3}(\partial_Z \mathcal{A}_0)^2}\,, \tag{3.7}$$

where

$$a \equiv \frac{N_c N_f \lambda^3 M_{\mathrm{KK}}^4}{3^9 \pi^5}, \qquad b \equiv \frac{3^6\pi^2}{4\lambda^2 M_{\mathrm{KK}}^2}. \tag{3.8}$$

Now we introduce the baryon source coupled to $\mathcal{A}_0$ through the Chern-Simons term [8, 5, 13] as mentioned before. We assume that baryons are uniformly distributed over $\mathbb{R}^3$ space

[2]The integral is extended to $(-\infty, \infty)$ to take into account $\overline{\mathrm{D8}}$ branes as well as D8 branes.



(a) (b)

Figure 1: (a) The profile of $\mathcal{A}_0(Z)$, (b) Chemical potential vs baryon charge ($\frac{\mu}{m_\rho}$ vs $\frac{Q}{n_0}$, where $Q \equiv n_q/2$).

whose volume is V. For large λ, the instanton size is $1/\sqrt{\lambda}$ [5, 14]. It can be treated as a static delta function source at large N_c. For a uniform baryon distribution, the source is

$$S_{\text{source}} = N_c n_B \int d^4x \int dZ\, \delta(Z)\mathcal{A}_0(Z). \tag{3.9}$$

The equation of motion of $\mathcal{A}_0$ is

$$\frac{d}{dZ}\frac{\partial\mathcal{L}}{\partial(\partial_Z\mathcal{A}_0)} = n_q\delta(Z)\,, \tag{3.10}$$

which yields

$$\frac{\partial\mathcal{L}}{\partial(\partial_Z\mathcal{A}_0)} = \frac{1}{2}n_q\ \text{sgn}(Z)\,, \tag{3.11}$$

where $n_q = N_c n_B$ is the quark density and the step function sig(Z) is determined by the symmetry between D8 ($Z > 0$) and $\overline{\text{D8}}$($Z < 0$). By integrating once more we get the classical solution $\mathcal{A}_0$

$$\mathcal{A}_0(Z;n_q) = \mathcal{A}_0(0) + \int_0^Z dZ \frac{n_q/2}{\sqrt{(ab)^2K^2 + bK^{1/3}n_q^2/4}}\ . \tag{3.12}$$

We introduce the "baryon charge chemical potential of a quark", μ, by [10, 12]

$$\mu(n_q) \equiv \lim_{|Z|\to\infty} \mathcal{A}_0(Z;n_q)\ . \tag{3.13}$$

This relation also defines μ as a function of n_q and vice versa. Furthermore we define the baryon chemical potential as

$$\mu_B = m_B + N_c\mu\ . \tag{3.14}$$

In figure (1a) we plot the profile of $\mathcal{A}_0(Z)$ in the Z coordinate and in figure (1b) we show μ for various baryon densities. Since we work in the canonical formalism μ is more like a Lagrange constraint.

Throughout, the numerics will be carried using the following values [3, 4]: $N_f = 2$, $N_c = 3$, $f_\pi = 92.6$MeV, and $m_\rho = 776$MeV. The smallest eigenvalue was calculated to be $\lambda_1 = 0.669$. Using these five values we can estimate $M_{\rm KK}$, λ, κ, a, and b:

$$M_{\rm KK} = \frac{m_\rho}{\sqrt{\lambda_1}} \simeq 950\, MeV, \quad \lambda \equiv g_{YM}^2 N_c = f_\pi^2 \frac{54\pi^4}{N_c M_{\rm KK}^2} \simeq 16.71, \quad \kappa \equiv \frac{\lambda N_c}{216\pi^3} \simeq 0.0075, \tag{3.15}$$

and

$$a = 3.76 \cdot 10^9\, MeV^4, \quad b = 7.16 \times 10^{-6} MeV^{-2}. \tag{3.16}$$

The definition of κ and λ are different from [3, 4] by a factor of 2, but it is consistent with [5]. In all figures n_B is normalized to $\frac{n_B}{n_0}$, with n_0 the nuclear matter density,

$$n_0 = 0.17\, fm^{-3} \simeq 1.3 \times 10^6\, MeV^3. \tag{3.17}$$

4. Thermodynamics

Consider the action (3.7) with the source term (3.10),

$$S = \int d^4x \int_{-\infty}^{+\infty} dZ\, \mathcal{L}$$

$$\text{with} \quad \mathcal{L} \equiv -aK^{2/3}\sqrt{1 - bK^{1/3}(\partial_Z \mathcal{A}_0)^2} + n_q \delta(Z)\mathcal{A}_0(Z)\ . \tag{4.1}$$

The $\mathcal{A}_0$ is an auxillary field with no time-dependence. It can be eliminated by the equation of motion (3.11) and (3.12). The energy is

$$\begin{aligned} U(n_q) &= \int dx^3 \int_{-\infty}^{+\infty} dZ\, (-\mathcal{L}) \\ &= aV \int_{-\infty}^{+\infty} dZ\, K^{2/3}\sqrt{1 + \frac{n_q^2}{4a^2 b} K^{-5/3}} - n_q \mu\,, \end{aligned} \tag{4.2}$$

where V is short for $\int dx^3$ and we may set $\mu = 0$. The chemical potential μ is constrained by the Gibbs relation $\mu = \frac{\partial F(n_q)}{\partial n_q}$ where $F(n_q)$ is the Helmholtz free energy which is $\mathrm{U}(n_q)$ at zero temperature. Thus

$$\mu = \int_{-\infty}^{\infty} dZ \frac{n_q/4}{\sqrt{(ab)^2 K^2 + bK^{1/3} n_q^2/4}}\ , \tag{4.3}$$

which is in agreement with the solution (3.12) for $A_0(0) = 0$. We note that this construction is consistent with [8, 9, 11, 10] where the grand potential is identified with the DBI action at finite μ.

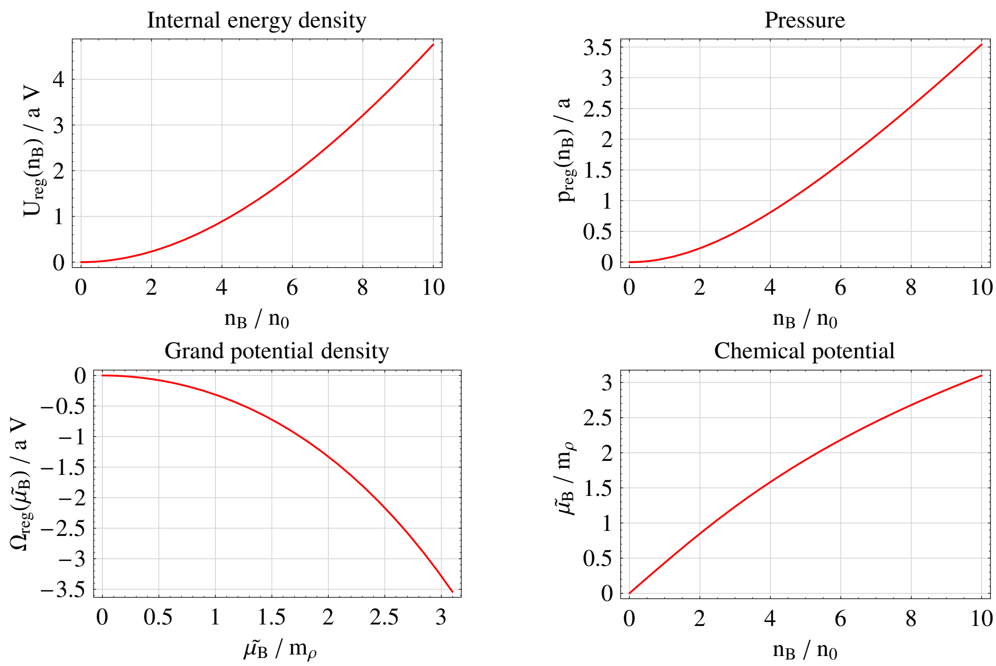


Figure 2: Numerical behaviour of the thermodynamic functions: See eq. (4.5)

In terms of the baryon number density n_B (n_q/N_c) the regularized Helmholtz free energy is

$$\frac{F_{\rm reg}(n_B)}{aV} \equiv \int_{-\infty}^{\infty} dZ K^{2/3}\left[\sqrt{1+\frac{(N_c n_B)^2}{4a^2 b}K^{-5/3}}-1\right] , \tag{4.4}$$

after subtracting the vacuum value. The regularized internal energy U, pressure p and grand potential Ω as a function of baryon number density n_B or the baryon chemical potential μ_B are

$$\begin{aligned}
\frac{U_{\rm reg}(n_B)}{aV} &= \int_{-\infty}^{\infty} dZ K^{2/3}\left[\sqrt{1+\frac{(N_c n_B)^2}{4a^2 b}K^{-5/3}}-1\right] , \\
\frac{p(n_B)_{\rm reg}}{a} &= \int_{-\infty}^{\infty} dZ\, K^{2/3}\left[1-\frac{1}{\sqrt{1+\frac{(N_c n_B)^2}{4a^2 b}K^{-5/3}}}\right] , \\
\frac{\Omega_{\rm reg}(\widetilde{\mu_B})}{aV} &= \int_{-\infty}^{\infty} dZ\, K^{2/3}\left[\frac{1}{\sqrt{1+\frac{(N_c n_B(\widetilde{\mu_B}))^2}{a^2 b}K^{-5/3}}}-1\right] , \\
\widetilde{\mu_B} &= N_c \int_{-\infty}^{\infty} dZ \frac{N_c n_B/4}{\sqrt{(ab)^2 K^2 + bK^{1/3}(N_c n_B/2)^2}} ,
\end{aligned} \tag{4.5}$$

where $\widetilde{\mu_B} \equiv \mu_B - m_B = N_c \mu$.

In figure (2) we present the numerical plots of these thermodynamic functions with the numerical inputs in section 3. For small baryon densities the energy density is quadratic

Thermodynamic function	$n_B/n_0 \sim 0$	$n_B/n_0 \sim 10$	$n_B/n_0 \to \infty$
Internal energy	$(n_B/n_0)^2$	$(n_B/n_0)^{1.85}$	$(n_B/n_0)^{1.4}$
Pressure	$(n_B/n_0)^2$	$(n_B/n_0)^{1.45}$	$(n_B/n_0)^{1.4}$
Chemical potential	$(n_B/n_0)^1$	$(n_B/n_0)^{0.67}$	$(n_B/n_0)^{0.4}$
Grand potential	$-(\widetilde{\mu_B}/m_\rho)^2$	$-(\widetilde{\mu_B}/m_\rho)^{2.16}$	$-(\widetilde{\mu_B}/m_\rho)^{3.5}$

Table 1: Numerical behaviour of the thermodynamic functions: See eq. (4.5)

in n_B/n_0 (or μ/m_ρ). At large baryon densities it is of order $(n_B/n_0)^{1.4}$. The small density limit can be qualitatively understood by noting that in bulk the $\mathcal{A}_0$ configuration for fixed charge is obtained by minimizing the induced DBI action of D8-$\overline{\text{D8}}$. Thus only flavor-meson mediated interactions between the point-like baryons are included. At large N_c the $D4$ mediated correlated gravitons (glueballs on the boundary) are heavy and decouple. Since our point baryonic vertices in bulk map on infinite size skyrmions at the boundary this implies that only ω exchanges survive at large N_c. Rho and pion exchange relies on skyrmion gradients which are zero. At low baryon densities, the dominant Skyrmion-omega-Skyrmion interaction is two-body and repulsive. Thus the energy density is positive and quadratic in the baryon density. The baryonic matter is prevented from flying apart by the container V. At large baryon densities, the energy density softens as the quark chemical potential is seen to saturate to $(n_B/n_0)^{0.4}$ numerically. We recall that the baryons are fixed sources so no Fermi motion is involved to this order. The pressure behaves as $(n_B/n_0)^2$ at low baryon densities, and again softens to $(n_B/n_0)^{7/5}$ at large baryon densities from the plot. We summarize the behaviour of the thermodynamic functions obtained numerically in table. (1). In this paper we do not consider the back reaction of gravity for baryons or D8 brane, therefore the behaviour at higher densities, say $n_B/n_0 \gg 10$, is not justified.

5. Effective meson action: $n_B = 0$

In [3, 4] the meson spectrum and coupling was studied at zero baryon density by analyzing the DBI action of D8-$\overline{\text{D8}}$ branes with the fluctuating gauge field A_M. We want to extend the analysis to *finite baryon density* or $n_B \neq 0$. For this purpose we streamline in this section the construction in [3, 4] for notational purposes and completeness. In the next two sections we add the background $\mathrm{U}(1)_V$ field $\mathcal{A}_0$ to the fluctuating gauge field A_M. It will enable us to study meson properties at finite baryon density.

5.1 Mode decomposition of A_M

The gauge field A_M has nine components, $A_\mu = A_{1,2,3,4}$, $A_z(\equiv A_5)$, and $A_\alpha(\alpha = 5,6,7,8,$ the coordinates on the S^4). We assume that $A_\alpha = 0$, and A_μ and A_z are independent of the coordinate on S^4. We further assume that A_M can be expanded in terms of complete sets, $\psi_n(z)$ and $\phi_n(z)$ as

$$A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu)\psi_n(z)\,, \tag{5.1}$$

$$A_z(x^\mu, z) = \varphi^{(0)}(x^\mu)\phi_0(z) + \sum_{n=1}^{\infty} \varphi^{(n)}(x^\mu)\phi_n(z)\,, \tag{5.2}$$

where $B_\mu^{(n)}$ is identified with vector and axial vector mesons and $\varphi^{(0)}$ with pions. $\varphi^{(n)}$ can be absorbed into $B_\mu^{(n)}$ through the gauge transformation (section 5.2). ψ_n satisfies the eigenvalue equation,

$$-K^{1/3}\,\partial_Z\left(K\,\partial_Z\psi_n\right) = \lambda_n\psi_n\,, \tag{5.3}$$

with the boundary condition $\partial_Z\psi_n(0) = 0$ (vector meson) or $\psi_n(0) = 0$ (axial vector meson) at $Z = 0$. They are normalized by

$$\kappa \int dZ\, K^{-1/3}\psi_n\psi_m = \delta_{nm}\,, \tag{5.4}$$

where $\kappa \equiv \widetilde{T}(2\pi\alpha')^2 R^3 = \frac{\lambda N_c}{216\pi^3}$, and (5.3) and (5.4) implies

$$\kappa \int dZ\, K(\partial_Z\psi_n)(\partial_Z\psi_m) = \lambda_n\delta_{nm}\,. \tag{5.5}$$

The $\phi_n(Z)$ are chosen such that

$$\phi_n(Z) = \frac{1}{\sqrt{\lambda_n}M_{\rm KK}U_{\rm KK}}\partial_Z\psi_n(Z) \quad (n \geq 1)\,, \tag{5.6}$$

$$\phi_0(Z) = \frac{1}{\sqrt{\pi\kappa}M_{kk}U_{\rm KK}}\frac{1}{K}\,, \tag{5.7}$$

with the normalization condition:

$$(\phi_m, \phi_n) \equiv \kappa M_{\rm KK}^2 U_{\rm KK}^2 \int dZ\, K\,\phi_m\phi_n = \delta_{mn}\,, \tag{5.8}$$

which is compatible with (5.5).

5.2 Effective meson action

With the gauge field $A_\mu(x^\mu, z)$ and $A_z(x^\mu, z)$ the DBI action of the D8-$\overline{\rm D8}$-branes becomes 5-dimensional [3]:

$$S^{\rm DBI}_{\rm D8\text{-}\overline{D8}} = -\widetilde{T}\int d^4x dz\, U^2 \tag{5.9}$$
$$\mathrm{tr}\,\sqrt{1 + (2\pi\alpha')^2\frac{R^3}{2U^3}F_{\mu\nu}F^{\mu\nu} + (2\pi\alpha')^2\frac{9}{4}\frac{U}{U_{\rm KK}}F_{\mu z}F^{\mu z} + [F^3] + [F^4] + [F^5]}\,,$$

where $\widetilde{T} = \frac{N_c}{216\pi^5}\frac{M_{\rm KK}}{\alpha'^3}$, U is a function of z by (3.4), and the indices are contracted by the metric $(-,+,+,+,+)$. $[F^3], [F^4]$, and $[F^5]$ are short for the terms of F^3, F^4, and F^5

[3] The gauge group generators t^a are normalized as $\mathrm{tr}\, t^a t^b = \delta_{ab}/2$

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respectively. Notice that the range of z is extended from $[0,\infty]$ to $[-\infty,\infty]$ to account for both $D8$ and $\overline{\mathrm{D8}}$.

Inserting (5.1) and (5.2) into (5.9) and using the orthonomality of ψ_n and ϕ_n ((5.4)$\sim$(5.8)), we have [3, 4]

$$S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}} \sim \int d^4x \,\mathrm{tr}\left[(\partial_\mu\varphi^{(0)})^2 + \sum_{n=1}^{\infty}\left(\frac{1}{2}(\partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)})^2 \right.\right.$$
$$\left.\left. +\lambda_n M_{\mathrm{KK}}^2(B_\mu^{(n)} - \lambda_n^{-1/2}\partial_\mu\varphi^{(n)})^2\right)\right]$$
$$+(\text{interaction terms})\ . \tag{5.10}$$

Here $\varphi^{(0)}$ and $B_\mu^{(n)}$ are interpreted as a masseless pion field and an infinite tower of vector (or axial) vector meson fields with masses $m_n^2(\equiv \lambda_n M_{\mathrm{KK}}^2)$. The lightest vector meson ρ is identified with $B_\mu^{(1)}$. $\varphi^{(n)}$ are absorbed into $B_\mu^{(n)}$. In the expansion (5.1) and (5.2), we have implicitly assumed that the gauge fields are zero asymptotically, i.e. $A_M(x^\mu,z)\to 0$ as $z\to\pm\infty$. The residual gauge transformation that does not break this condition is obtained by a gauge function $g(x^\mu,z)$ that asymptotes a constant $g(x^\mu,z)\to g_\pm$ at $z\pm\infty$. (g_+,g_-) are interpreted as elements of the chiral symmetry group $\mathrm{U}(N_f)_L\times\mathrm{U}(N_f)_R$ in QCD with N_f massless flavors.

5.3 $A_z = 0$ gauge and pion effective action

In the previous subsection we worked in the gauge $A_M(x^\mu,z)\to 0$ as $z\to\pm\infty$. However the $A_z=0$ gauge can be achieved by applying the gauge transformation $A_M\to gA_Mg^{-1} + g\partial_M g^{-1}$ with the gauge function

$$g^{-1}(x^\mu,z) = P\exp\left\{-\int_0^z dz'\, A_z(x^\mu,z')\right\}\ . \tag{5.11}$$

Then the asymptotic values of $A_\mu(z\to\infty)$ do not vanish and change to

$$A_\mu(x^\mu,z)\to \xi_\pm(x^\mu)\partial_\mu\xi_\pm^{-1}(x^\mu) \qquad \text{as}\quad z\to\pm\infty\,, \tag{5.12}$$

where $\xi_\pm(x^\mu)\equiv \lim_{z\to\pm\infty} g(x^\mu,z)$. The gauge fields can be expanded as

$$A_\mu(x^\mu,z) = \xi_+(x^\mu)\partial_\mu\xi_+^{-1}(x^\mu)(x^\mu)\psi_+(z) + \xi_-(x^\mu)\partial_\mu\xi_-^{-1}(x^\mu)\psi_-(z) + \sum_{n=1}^\infty B_\mu^{(n)}(x^\mu)\psi_n(z)\,,$$
$$A_z(x^\mu,z) = 0\,, \tag{5.13}$$

where $\psi_\pm$ is the non-normalizable zero mode of (5.3) with the appropriate boundary condition to yield (5.12):

$$\psi_\pm = \frac{1}{2}\pm\widehat{\psi}_0\,,$$
$$\widehat{\psi}_0 = \frac{1}{\pi}\arctan(Z) \tag{5.14}$$

There is a residual gauge symmetry which maintains $A_z = 0$. It is given by the z-independent gauge transformation $h(x^\mu)$,

$$A_M(x^\mu, z) \to h(x^\mu) A_M(x^\mu, z) h^{-1}(x^\mu) + h(x^\mu)\partial_M h^{-1}(x^\mu)\,, \tag{5.15}$$

which acts on the component fields as

$$\xi_\pm \;\to\; h\,\xi_\pm\, g_\pm^{-1}\,, \tag{5.16}$$
$$B_\mu^{(n)} \;\to\; h\, B_\mu^{(n)}\, h^{-1}\,, \tag{5.17}$$

where we considered chiral symmetry $g_\pm$ together. Then $\xi_\pm(x^\mu)$ are interpreted as the $\mathrm{U}(N_f)$ valued fields $\xi_{L,R}(x^\mu)$ which carry the pion degrees of freedom in the hidden local symmetry approach . Indeed the transformation property (5.16) is the same as that for $\xi_{L,R}(x^\mu)$ if we interpret $h(x^\mu) \in \mathrm{U}(N_f)$ as the hidden local symmetry. They are related to the $\mathrm{U}(N_f)$ valued pion field $\mathrm{U}(x^\mu)$ in the chiral Lagrangian by

$$\xi_+^{-1}(x^\mu)\xi_-(x^\mu) = \mathrm{U}(x^\mu) \equiv e^{2i\Pi(x^\mu)/f_\pi}\;. \tag{5.18}$$

The pion field $\Pi(x^\mu)$ is identical to $\varphi^{(0)}(x^\mu)$ in (5.2) in leading order. A convenient gauge choice is

$$\xi_-(x^\mu) = 1, \quad \xi_+^{-1}(x^\mu) = \mathrm{U}(x^\mu) = e^{2i\Pi(x^\mu)/f_\pi} \tag{5.19}$$

which expresses the gauge fields as,

$$A_\mu(x^\mu, z) = U^{-1}(x^\mu)\partial_\mu\, \mathrm{U}(x^\mu)\psi_+(z) + \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z) \tag{5.20}$$

In this gauge, after omitting the vector meson fields $B_\mu^{(n)}$, the effective action reduces to the Skyrme model

$$S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}}\Big|_{B_\mu^{(n)}=0} = \int d^4x \left(\frac{\kappa M_{\mathrm{KK}}^2}{\pi} \mathrm{tr}\, \left(U^{-1}\partial_\mu U\right)^2 + \frac{1}{32 e_S^2} \mathrm{tr}\, \left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]^2 \right), \tag{5.21}$$

where $e_S^{-2} \equiv \kappa \int dz\, K^{-1/3}(1-\psi_0^2)^2$ and the pion decay constant f_π is fixed by the comparison with the Skyrme model:

$$f_\pi^2 \equiv \frac{4}{\pi}\kappa M_{\mathrm{KK}}^2 = \frac{1}{54\pi^4} M_{\mathrm{KK}}^2 \lambda N_c\,, \tag{5.22}$$

Another gauge we will consider below is

$$\xi_+^{-1}(x^\mu) = \xi_-(x^\mu) = e^{i\Pi(x^\mu)/f_\pi}\;. \tag{5.23}$$

in terms of which the gauge fields are written as

$$A_\mu(x^\mu, z) \;=\; \alpha_\mu(x^\mu)\widehat{\psi}_0(z) + \beta_\mu(x^\mu) + \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu)\psi_n(z)\,, \tag{5.24}$$
$$\alpha_\mu(x^\mu) \;=\; \{\xi^{-1}, \partial_\mu \xi\} = \frac{2i}{f_\pi}\partial_\mu \Pi + [[\partial_\mu \Pi^3]] + \mathcal{O}(\Pi^4)\,,$$
$$\beta_\mu(x^\mu) \;=\; \frac{1}{2}[\xi^{-1}, \partial_\mu \xi] = \frac{1}{2f_\pi^2}[\Pi, \partial_\mu \Pi] + \mathcal{O}(\Pi^4)\,,$$

where $[[\partial_\mu \Pi^3]] \equiv -\frac{i}{3f_\pi^3}((\partial_\mu \Pi)\Pi^2 + \Pi^2 \partial_\mu \Pi - 2\Pi(\partial_\mu \Pi)\Pi)$.

6. Effective meson action: $n_B \neq 0$

We now extend the previous analysis to finite baryon density for $n_B = 0$. This is achieved by adding the background $\mathrm{U}(1)_V$ field $\mathcal{A}_0$ to the fluctuating gauge field A_M. Since, the vacuum modes $\{\psi_n, \phi_n\}$ are not mass eigenmodes in matter, we may choose more pertinent eigenmodes in matter. Two basis set are possible: (1) medium mass eigenmodes $\psi_n \sim e^{-imt} f_n(z)$; (2) screening eigenmodes $\psi \sim e^{i\vec{k}\cdot\vec{x}} f_n(z)$. With this in mind, we have the following gauge fields decomposition

$$A_0(x^\mu, z) = \mathcal{A}_0(z) + \sum_{n=1}^{\infty} B_0^{(n)}(x^\mu)\omega_n(z)\,, \tag{6.1}$$

$$A_i(x^\mu, z) = \sum_{n=1}^{\infty} B_i^{(n)}(x^\mu)\psi_n(z)\,, \tag{6.2}$$

$$A_z(x^\mu, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x^\mu)\phi_n(z)\,. \tag{6.3}$$

$\mathcal{A}_0(z)$ is the background gauge field. The time component modes ($\omega_n(z)$) and the space component ($\psi_n(z)$) are not necessarily the same as Lorentz symmetry does not hold in the matter rest frame. Note that $F_{\mu z}$ is modified by $\mathcal{A}_0$ while $F_{\mu\nu}$ is not.

In order to compute the DBI action (5.9),

$$S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}} = -\widetilde{T}\int d^4x dz\, U^2$$
$$\mathrm{tr}\,\sqrt{1 + (2\pi\alpha')^2 \frac{R^3}{2U^3} F_{\mu\nu}F^{\mu\nu} + (2\pi\alpha')^2 \frac{9}{4}\frac{U}{U_{\mathrm{KK}}} F_{\mu z}F^{\mu z} + [F^3] + [F^4] + [F^5]}\,,$$

we need to know $F_{\mu\nu}F^{\mu\nu}, F_{\mu z}F^{\mu z}, [F^3], [F^4]$, and $[F^5]$, which are involved in general. To quadratic order (ignoring $\mathcal{O}((B_\mu, \varphi)^3)$), the contributions are greatly simplified because of: 1) cyclic property of the trace, 2) antisymmetry of $F_{\mu,\nu}$, 3) parity of mode functions. Then there is no contribution from $[F^3]$ and $[F^5]$. $[F^4]$ has important terms that will modify $F_{\mu\nu}F^{\mu\nu}$:

$$[F^4] = (2\pi\alpha')^4 \frac{9}{8}\frac{U}{U_{\mathrm{KK}}}\left(\frac{R}{U}\right)^3 F_{0z}F^{0z}F_{ij}F^{ij} + \mathcal{O}((B_\mu, \varphi)^4)\,. \tag{6.4}$$

Table (2) lists all the relevant terms, where we have introduced f_{ij} defined as

$$f_{ij} \equiv \partial_i v_j - \partial_j v_i\,, \tag{6.5}$$

with $i, j = 1, 2, 3$. Table (2) should be understood in the integral and trace operation. We omitted some terms vanishing in those operations and rearranged some terms by using the cyclicity of the trace.

In terms of the definitions on the r.h.s. of the table (2) , the action reads

$$S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}} = -\widetilde{T}\int d^4x dz\, U^2\, \mathrm{tr}\,\sqrt{P_0 + P_1}\,, \tag{6.6}$$

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$F_{\mu\nu}F^{\mu\nu}$	$\to$	$\Big[\ 2\partial_0 B_i^{(n)}\partial^0 B^{(m)i}\psi_n\psi_m + 2\partial_i B_0^{(n)}\partial^j B^{(m)0}\omega_n\omega_m$		
		$-2\partial_0 B_i^{(n)}\partial^i B^{(m)0}\psi_n\psi_m$		
		$+(\partial_i B_j^{(n)} - \partial_j B_i^{(n)})(\partial^i B^{(m)j} - \partial^j B^{(m)i})\psi_n\psi_m\ \Big]$	$\equiv$	α_2
$F_{\mu z}F^{\mu z}$	$\to$	$-(\dot{\mathcal{A}}_0)^2$	$\equiv$	β_0
		$+2\dot{\mathcal{A}}_0\Big[\partial^0\varphi^{(n)}\phi_n - B^{0(n)}\dot{\omega}_n + [B^{(n)0}, \varphi^{(m)}]\omega_n\phi_m\Big]$	$\equiv$	β_1
		$+\Big[\partial_0\varphi^{(n)}\partial^0\varphi^{(m)}\phi_n\phi_m + B_0^{(n)}B^{(m)0}\dot{\omega}_n\dot{\omega}_m - 2\partial_0\varphi^{(n)}B^{(m)0}\phi_n\dot{\omega}_m$		
		$+\partial_i\varphi^{(n)}\partial^i\varphi^{(m)}\phi_n\phi_m + B_i^{(n)}B^{(m)i}\dot{\psi}_n\dot{\psi}_m - 2\partial_i\varphi^{(n)}B^{(m)i}\phi_n\dot{\psi}_m\Big]$	$\equiv$	β_2
$[F^4]$	$\to$	$f_{ij}f^{ij}(\dot{\mathcal{A}}_0)^2\psi_1^2$	$\equiv$	γ_2

Table 2: The relevant terms in evaluating the DBI action up to quadratic order in the fields (B_μ, φ). The upper dot stands for the derivative with respect to z. The terms should be understood in the integral and trace operation.

with

$$P_0 \equiv 1 - (2\pi\alpha')^2 \frac{9}{4}\frac{U}{U_{\mathrm{KK}}}\ \beta_0 = 1 - bK^{\frac{1}{3}}(\partial_Z \mathcal{A}_0)^2\,, \tag{6.7}$$

$$P_1 \equiv (2\pi\alpha')^2 \frac{R^3}{2U^3}(\alpha_2) + (2\pi\alpha')^2 \frac{9}{4}\frac{U}{U_{\mathrm{KK}}}(\beta_1 + \beta_2) + (2\pi\alpha')^4 \frac{9}{8}\frac{R^3}{U_{\mathrm{KK}}U^2}(\gamma_2)\,, \tag{6.8}$$

where P_0 does not contain meson fields but involves the baryon density. Expanding the action for small fields we have

$$\begin{aligned} S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}} &= -\widetilde{T}\int d^4x dz\, U^2\, \mathrm{tr}\ \left[\sqrt{P_0} + \frac{1}{2}\frac{P_1}{\sqrt{P_0}} - \frac{1}{8}\frac{P_1^2}{\sqrt{P_0}^3}\right] + \mathcal{O}((B_\mu,\varphi)^3) \\ &= S_1 + S_2 + \mathcal{O}((B_\mu,\varphi)^3)\,, \end{aligned} \tag{6.9}$$

with

$$S_1 \equiv -\widetilde{T}\int d^4x dz\, U^2 \mathrm{tr}\ \Delta^{-1}\,, \tag{6.10}$$

$$S_2 \equiv -\widetilde{T}\int d^4x dz\, U^2 \mathrm{tr}\ \left[\frac{1}{2}\Delta P_1 - \frac{1}{8}\Delta^3 P_1^2\right]\,, \tag{6.11}$$

where the modification factor $\Delta(n_B)$ is

$$\Delta(n_B) \equiv \frac{1}{\sqrt{P_0}} = \frac{1}{\sqrt{1 - bK^{\frac{1}{3}}(\partial_Z \mathcal{A}_0)^2}} = \sqrt{1 + \frac{n_B^2}{4a^2 b}K^{-5/3}}\,.$$

$-S_0$ is the grand potential discussed in section 4, and S_2 will be reduced to [4]

$$
\begin{aligned}
S_2 = -\mathrm{tr} \int d^4x \Bigg\{ & \left[\int dZ K^{-1/3}\Delta\ \Psi_n\Psi_m\right] \partial_0 B_i^{(m)} \partial^0 B^{(n)i} \qquad (6.12)\\
&+ \left[\int dZ K^{-1/3}\Delta\ \Omega_n\ \Omega_m\right] \partial_i B_0^{(n)} \partial^i B^{(m)0} - \left[\int dZ K^{-1/3}\Delta\ \Psi_n\ \Omega_m\right] 2\partial_0 B_i^{(n)} \partial^i B^{(m)0}\\
&+ \left[\int dZ K^{-1/3}\Delta^{-1}\Psi_n\Psi_m\right] \frac{1}{2}(\partial_i B_j^{(n)} - \partial_j B_i^{(n)})(\partial^i B^{(m)j} - \partial^j B^{(m)i})\\
&+ \left[M_{\mathrm{KK}}^2 \int dZ K\Delta^3\ \partial_Z\Omega_n\ \partial_Z\Omega_m\right] B_0^{(n)} B^{(m)0}\\
&+ \left[M_{\mathrm{KK}}^2 \int dZ K\Delta\ \partial_Z\Psi_n\partial_Z\Psi_m\right] B_i^{(n)} B^{(m)i}\\
&+ \left[M_{\mathrm{KK}}^2 \int dZ K\Delta^3\Phi_n\Phi_m\right] \partial_0\varphi^{(n)}\partial^0\varphi^{(m)} + \left[M_{\mathrm{KK}}^2 \int dZ K\Delta\Phi_n\Phi_m\right] \partial_i\varphi^{(n)}\partial^i\varphi^{(m)}\\
&- \left[M_{\mathrm{KK}}^2 \int dZ K\Delta^3\Phi_n\partial_Z\Omega_m\right] 2\partial_0\varphi^{(n)} B^{(m)0}\\
&- \left[M_{\mathrm{KK}}^2 \int dZ K\Delta\Phi_n\partial_Z\Psi_m\right] 2\partial_i\varphi^{(n)} B^{(m)i} \Bigg\},
\end{aligned}
$$

where we defined the scaled eigenfounctions as

$$
\Omega_n \equiv \sqrt{\kappa}\omega_n\,, \quad \Phi_n \equiv \sqrt{\kappa}\psi_n\,, \quad \Phi_n \equiv \sqrt{\kappa}U_{\mathrm{KK}}\phi_n\ . \qquad (6.13)
$$

At zero density $\Delta = 1$, so $\Phi_n = \Omega_n$ and the action reduces to the (5.10) by the same mode function in (5.3) $\sim$ (5.8). However at finite density the eigen modes Ω_n, Ψ_n, and Φ_n cannot be determined uniquely. In other words there is no mode decomposition which makes the action completly diagonal. So we consider the space-like and time-like separatly: (1) $A_M = A_M(x^i, z)$ and (2) $A_M = A_M(x^0, z)$.

6.1 Space-like fields $A_M = A_M(x^i, z)$

First we consider time-independent gauge fields. Up to quadratic order the action is

$$
\begin{aligned}
S_2 = -\mathrm{tr} \int d^4x \Bigg\{ & \left[\int dZ K^{-1/3}\Delta\ \Omega_n\ \Omega_m\right] \partial_i B_0^{(n)} \partial^i B^{(m)0} \qquad (6.14)\\
&+ \left[\int dZ K^{-1/3}\Delta^{-1}\Psi_n^S\Psi_m^S\right] \frac{1}{2}(\partial_i B_j^{(n)} - \partial_j B_i^{(n)})(\partial^i B^{(m)j} - \partial^j B^{(m)i})\\
&+ \left[M_{\mathrm{KK}}^2 \int dZ K\Delta^3\ \partial_Z\Omega_n\ \partial_Z\Omega_m\right] B_0^{(n)} B^{(m)0}\\
&+ \left[M_{\mathrm{KK}}^2 \int dZ K\Delta\ \partial_Z\Psi_n^S\partial_Z\Psi_m^S\right] B_i^{(n)} B^{(m)i}\\
&+ \left[M_{\mathrm{KK}}^2 \int dZ K\Delta\Phi_n^S\Phi_m^S\right] \partial_i\varphi^{(n)}\partial^i\varphi^{(m)} - \left[M_{\mathrm{KK}}^2 \int dZ K\Delta\Phi_n^S\partial_Z\Psi_m^S\right] 2\partial_i\varphi^{(n)} B^{i(m)} \Bigg\},
\end{aligned}
$$

[4]Note that the pattern: Δ, Δ^{-1}, and Δ^3. This pattern appears also when we consider higher order terms including couplings. The origin is explained in appendix A.

where we have defined the scaled eigenfunctions as

$$\Omega_n \equiv \sqrt{\kappa}\omega_n\,, \quad \Psi_n^S \equiv \sqrt{\kappa}\psi_n\,, \quad \Phi_n^S \equiv \sqrt{\kappa}U_{\rm KK}\phi_n\ . \tag{6.15}$$

To diagonalize the action we choose Ψ_n^S as the eigenfunction satisfying

$$-K^{1/3}\Delta^{-1}\partial_Z\left(K\Delta^3\,\partial_Z\Omega_n\right) = \lambda_n^\Omega\Omega_n\,, \tag{6.16}$$
$$-K^{1/3}\Delta\,\partial_Z\left(K\Delta\,\partial_Z\Psi_n^S\right) = \lambda_n^S\Psi_n^S\,, \tag{6.17}$$

with the normalization conditions,

$$\int dZ K^{-1/3}\Delta\,\Omega_n\,\Omega_m = \delta_{nm}\,, \tag{6.18}$$
$$\int dZ K^{-1/3}\Delta^{-1}\Psi_n^S\Psi_m^S = \delta_{nm}\,, \tag{6.19}$$

which imply

$$\int dZ K\Delta^3\,\partial_Z\Omega_n\,\partial_Z\Omega_m = \lambda_n^\Omega\delta_{nm}\,, \tag{6.20}$$
$$\int dZ K\Delta\,\partial_Z\Psi_n^S\partial_Z\Psi_m^S = \lambda_n^S\delta_{nm}\ . \tag{6.21}$$

If we choose Φ_n^S as

$$\Phi_n^S = \frac{1}{M_{\rm KK}\sqrt{\lambda_n^S}}\partial_Z\Psi_n^S\ \ (n\geq 1)\ , \quad \Phi_0^S = \frac{1}{M_{\rm KK}}\frac{1}{\sqrt{\int dZ(K^{-1}\Delta^{-1})}}\,\frac{1}{K\Delta}\,, \tag{6.22}$$

then $\partial_i\varphi^{(n)}$ $(n\geq 1)$ can be absorbed into $B_i^{(n)}$ through the gauge transformation

$$B_i^{(n)} \to B_i^{(n)} + \frac{1}{M_{\rm KK}\sqrt{\lambda_n^S}}\partial_i\varphi^{(n)}\ . \tag{6.23}$$

These choices of mode functions reduces the action to

$$\begin{aligned} S_2 = -{\rm tr}\int d^4x\Big\{&\partial_i\varphi^{(0)}\partial^i\varphi^{(0)} \\ &+\partial_i B_0^{(n)}\partial^i B^{(n)0} + \frac{1}{2}f_{ij}^{(n)}f^{(n)ij} + \mathcal{M}_n^{\shortparallel 2}B_0^{(n)}B^{(n)0} + \mathcal{M}_n^{\perp\,2}B_i^{(n)}B^{(n)i}\Big\}\,, \end{aligned} \tag{6.24}$$

where we have defined longitudinal screening masses $\mathcal{M}_n^{\shortparallel}$ and transverse screening masses $\mathcal{M}_n^{\perp}$ as

$$\mathcal{M}_n^{\shortparallel} \equiv \sqrt{\lambda_n^\Omega}M_{\rm KK}\,, \qquad \mathcal{M}_n^{\perp} \equiv \sqrt{\lambda_n^S}M_{\rm KK}\ . \tag{6.25}$$

6.2 Time-like fields $A_M = A_M(x^0, z)$

For spacially homogeneous gauge fields the action reads

$$
\begin{aligned}
S_2 = -\mathrm{tr} \int d^4x \Big\{ & \left[\int dZ K^{-1/3} \Delta \ \Psi_n^T \Psi_m^T \right] \partial_0 B_i^{(m)} \partial^0 B^{(n)i} \\
& + \left[M_{\mathrm{KK}}^2 \int dZ K \Delta \ \partial_Z \Psi_n^T \partial_Z \Psi_m^T \right] B_i^{(n)} B^{(m)i} \\
& + \left[M_{\mathrm{KK}}^2 \int dZ K \Delta^3 \ \partial_Z \Omega_n \ \partial_Z \Omega_m \right] B_0^{(n)} B^{(m)0} \\
& + \left[M_{\mathrm{KK}}^2 \int dZ K \Delta^3 \Phi_n^\Omega \Phi_m^\Omega \right] \partial_0 \varphi^{(n)} \partial^0 \varphi^{(m)} \\
& - \left[M_{\mathrm{KK}}^2 \int dZ K \Delta^3 \Phi_n^\Omega \partial_Z \Omega_m \right] 2 \partial_0 \varphi^{(n)} B^{0(m)} \Big\} ,
\end{aligned}
\tag{6.26}
$$

where we have defined the scaled eigenfunctions

$$
\Omega_n \equiv \sqrt{\kappa} \omega_n \,, \quad \Psi_n^T \equiv \sqrt{\kappa} \psi_n \,, \quad \Phi_n^\Omega \equiv \sqrt{\kappa} U_{\mathrm{KK}} \phi_n \,. \tag{6.27}
$$

We choose Ψ_n^S as the eigenfunction satisfying

$$
-K^{1/3} \Delta^{-1} \partial_Z \left(K \Delta \ \partial_Z \Psi_n^T \right) = \lambda_n^T \Psi_n^T \,, \tag{6.28}
$$

$$
-K^{1/3} \Delta^{-1} \partial_Z \left(K \Delta^3 \ \partial_Z \Omega_n \right) = \lambda_n^\Omega \Omega_n \,, \tag{6.29}
$$

with the normalization conditions,

$$
\int dZ K^{-1/3} \Delta \ \Psi_n^T \Psi_m^T = \delta_{nm} \,, \tag{6.30}
$$

$$
\int dZ K^{-1/3} \Delta \ \Omega_n \ \Omega_m = \delta_{nm} \,, \tag{6.31}
$$

which imply

$$
\int dZ K \Delta \ \partial_Z \Psi_n^T \partial_Z \Psi_m^T = \lambda_n^T d_{nm} \,, \tag{6.32}
$$

$$
\int dZ K \Delta^3 \ \partial_Z \Omega_n \ \partial_Z \Omega_m = \lambda_n^\Omega \delta_{nm} \,, \tag{6.33}
$$

If we choose Φ_n^S as

$$
\Phi_n^\Omega = \frac{1}{M_{\mathrm{KK}} \sqrt{\lambda_n^\Omega}} \partial_Z \Psi_n^\Omega \,, \quad \Phi_0^\Omega = \frac{1}{M_{\mathrm{KK}}} \frac{1}{\sqrt{\int dZ (K^{-1} \Delta^{-3})}} \ \frac{1}{K \Delta^3} \,, \tag{6.34}
$$

then $\partial_0 \varphi^{(n)}$ $(n \geq 1)$ can be absorbed into $B_0^{(n)}$ through the gauge transformation

$$
B_0^{(n)} \to B_0^{(n)} + \frac{1}{M_{\mathrm{KK}} \sqrt{\lambda_n^\Omega}} \partial_0 \varphi^{(n)} \,. \tag{6.35}
$$

The action is reduced to

$$S_2 = -\mathrm{tr} \int d^4x \Big\{ \partial_0 \varphi^{(n)} \partial^0 \varphi^{(n)} + \partial_0 B_i^{(n)} \partial^0 B^{(n)i} + m_n^2 B_i^{(n)} B^{(n)i} + M_{\mathrm{KK}}^2 \lambda_n^{\Omega} B_0^{(n)} B^{(n)0} \Big\} , \tag{6.36}$$

where we have defined the mass

$$m_n = \sqrt{\lambda_n^T} M_{\mathrm{KK}} . \tag{6.37}$$

6.3 Pion effective action

In this subsection we work in the $A_z = 0$ gauge following the procedure in section 5.3.

6.3.1 Time-like field $A_M = A_M(x^0, z)$

First consider the case $A_M = A_M(x^0, z)$. By the gauge transformation $A_M \to g A_M g^{-1} + g \partial_M g^{-1}$ with the gauge function

$$g^{-1}(x^0, z) = P \exp \left\{ - \int_0^z dz' \, A_z(x^0, z') \right\} , \tag{6.38}$$

the gauge fields are rewritten as

$$A_0(x^0, z) = \mathcal{A}_0(z) + \xi_+(x^0) \partial_0 \xi_+^{-1}(x^0) \omega_+(z) + \xi_-(x^0) \partial_0 \xi_-^{-1}(x^0) \omega_-(z) , \tag{6.39}$$

$$A_i(x^0, z) = A_z(x^0, z) = 0 , \tag{6.40}$$

where we have omitted the vector mesons $B_\mu^{(n)}$. The $\omega_\pm$ are obtained as zero mode solutions of (6.29) satisfying the boundary condition for $A_0(x^0, z)$:

$$\omega_\pm(z) \equiv \frac{1}{2} \pm \frac{1}{\int dZ (K^{-1} \Delta^{-3})} \int_0^Z dZ \frac{1}{K \Delta^3} . \tag{6.41}$$

By using the residual gauge symmetry $h(x^\mu)$ (5.19) and (5.20) we may express the gauge field as

$$A_0(x^0, z) = \mathcal{A}_0 + U^{-1}(x^0) \partial_0 \, \mathrm{U}(x^0) \omega_+(z) . \tag{6.42}$$

The field strength is

$$F_{z\mu} = \dot{\mathcal{A}}_0 + U^{-1} \partial_0 U \widehat{\phi}_0^\omega(z) , \qquad F_{\mu\nu} = 0 , \tag{6.43}$$

where

$$\widehat{\phi}_0^\omega(z) \equiv \partial_z \omega_+(z) = \frac{1}{U_{\mathrm{KK}} \int dZ (K^{-1} \Delta^{-3})} \frac{1}{K \Delta^3} . \tag{6.44}$$

The action becomes

$$S_2 = \mathrm{tr} \int d^4x \left[\kappa M_{\mathrm{KK}}^2 \frac{1}{\int dZ K^{-1} \Delta^{-3}} \right] (U^{-1} \partial_0 U)^2 , \tag{6.45}$$

and we identify the time-like pion decay constant f_π^T as

$$f_\pi^{T2} = \frac{4 \kappa M_{\mathrm{KK}}^2}{\int dZ K^{-1} \Delta^{-3}} , \tag{6.46}$$

by comparison with the Skyrme model.

6.3.2 Space-like field $A_M = A_M(x^i, z)$

Similarly, we consider the case $A_M = A_M(x^i, z)$. Using the same gauge transformation we can work with the gauge fields,

$$\begin{aligned} A_i(x^i, z) &= \xi_+(x^i)\partial_i \xi_+^{-1}(x^i)\psi_+^S(z) + \xi_-(x^i)\partial_i \xi_-^{-1}(x^i)\psi_-^S(z)\,, \\ A_0(x^i, z) &= \mathcal{A}_0(z)\,, \\ A_z(x^i, z) &= 0\,, \end{aligned}$$

where $\psi_\pm^S$ are obtained as a zero mode solution of (6.19) satisfying the pertinent boundary condition of $A_i(x^i, z)$:

$$\psi_\pm^S(z) \equiv \frac{1}{2} \pm \frac{1}{\int dZ(K^{-1}\Delta^{-1})} \int_0^Z dZ \frac{1}{K\Delta} \tag{6.47}$$

Then the gauge field and the field strength in the gauge (5.19) are

$$\begin{aligned} A_0(x^0, z) &= \mathcal{A}_0 + U^{-1}(x^i)\partial_i \, \mathrm{U}(x^i)\psi_+^S(z) \\ F_{z\mu} &= \dot{\mathcal{A}}_0 + U^{-1}\partial_i U \widehat{\phi}_0^S(z)\,, \end{aligned} \tag{6.48}$$

where we do not consider $F_{\mu\nu}$ since we are interested in the kinetic part and

$$\widehat{\phi}_0^S(z) \equiv \partial_z \psi_+^S(z) = \frac{1}{U_{\mathrm{KK}} \int dZ(K^{-1}\Delta^{-1})} \frac{1}{K\Delta}\,. \tag{6.49}$$

The action is

$$S_2 = \mathrm{tr} \int d^4x \left[\kappa M_{\mathrm{KK}}^2 \frac{1}{\int dZ K^{-1}\Delta^{-1}} \right] (U^{-1}\partial_i U)^2\,, \tag{6.50}$$

and f_π^S is identified by

$$f_\pi^{S\,2} = \frac{4\kappa M_{\mathrm{KK}}^2}{\int dZ K^{-1}\Delta^{-1}}\,. \tag{6.51}$$

6.4 Vector mesons interactions

In this section we study the interactions of the fields $B_0^{(1)}$, $B_i^{(1)}$ and $\varphi^{(0)}$ corresponding to the lowest medium modes Ω_1, Ψ_1, and Φ_1. For simplicity, we use the following notation,

$$v_0 \equiv B_0^{(1)}\,, \quad v_i \equiv B_i^{(1)}\,, \quad \Pi \equiv \varphi^{(0)}\,. \tag{6.52}$$

The details of the computation are relegated to appendix A [5].

[5]Both in appendix A and this section, the vector meson field is considered as anti Hermitian. Although in appendix A, we are working with the vacuum modes instead of the medium modes, the conversion can be done by inspection using the formula tabulated in table (4), (5)

6.4.1 Time-like fields $A_M = A_M(x^0, z)$

$$S_2 = \mathrm{tr} \int d^4x \Big\{ -\partial_0\Pi\partial^0\Pi + \partial_0 v_i \partial^0 v^i + m_1^2 v_i v^i + M_{\mathrm{KK}}^2 \lambda_1^\Omega v_0 v^0 \\ -2g_{v\Pi^2}^T\, v_0[\Pi, \partial^0\Pi] + g_{v^3}^T\, 2\partial_0 v_i [v^0, v^i] + \cdots \Big\}, \qquad (6.53)$$

where the couplings can be read from (4), (5) in appendix A by substituting the vacuum mode functions by the medium mode functions

$$\begin{aligned} g_{v\Pi^2}^T &= \frac{1}{\sqrt{\kappa}} \frac{\int dZ\, \frac{\Omega_1}{K\Delta^3}}{\int dZ\, \frac{1}{K\Delta^3}}, \\ g_{v^3}^T &= \frac{1}{\sqrt{\kappa}} \int dZ\, K^{-1/3} \Omega_1 (\Psi_1^T)^2 \Delta\,. \end{aligned} \qquad (6.54)$$

6.4.2 Space-like fields $A_M = A_M(x^i, z)$

$$S_2 = \mathrm{tr} \int d^4x \Big\{ -\partial_i\Pi\partial^i\Pi + \partial_i v_0 \partial^i v^0 + \frac{1}{2} f_{ij} f^{ij} + \mathcal{M}_1^{\shortparallel\,2} v_0 v^0 + \mathcal{M}_1^{\perp\,2} v_i v^i \\ -2g_{v\Pi^2}^S\, v_i[\Pi, \partial^i\Pi] + g_{v^3}^S\, 2\partial_i v_0 [v^i, v^0] + \widetilde{g}_{v^3}^S\, f_{ij}[v^i, v^j] + \cdots \Big\}, \qquad (6.55)$$

where the couplings can be read from (4), (5) in appendix A, again by substituting the vacuum mode functions by the medium mode functions

$$\begin{aligned} g_{v\Pi^2}^S &= \frac{1}{\sqrt{\kappa}} \frac{\int dZ\, \frac{\Psi_1^S}{K\Delta}}{\int dZ\, \frac{1}{K\Delta}}, \\ g_{v^3}^S &= \frac{1}{\sqrt{\kappa}} \int dZ\, K^{-1/3} (\Omega_1)^2 \Psi_1^S \Delta^{-1}, \\ \widetilde{g}_{v^3}^S &= \frac{1}{\sqrt{\kappa}} \int dZ\, K^{-1/3} (\Psi_1^S)^3 \Delta^{-1}\,. \end{aligned} \qquad (6.56)$$

6.4.3 Zero density limit

To check the current mode decomposition used in this section, we take the zero baryon density limit. In this case, Lorentz symmetry is enforced and the action reads

$$S_2 = \mathrm{tr} \int d^4x \Big\{ -\partial_\mu\Pi\partial^\nu\Pi + \frac{1}{2} f_{\mu\nu} f^{\mu\nu} + m_1^2 v_\mu v^\nu \\ -2g_{v\Pi^2}\, v_\mu[\Pi, \partial^\mu\Pi] + g_{v^3}\, f_{\mu\nu}[v^\mu, v^\nu] + \cdots \Big\}, \qquad (6.57)$$

which is the same as eq. (5.40) in [3] except the $v\Pi\Pi$ coupling. The difference comes from the gauge choice. In [3] $A_z = 0$ gauge is used and we chose $A_M(z \to \infty) \to 0$. Since the difference is merely a gauge choice, physics will not be changed. However we will repeat the analysis of couplings at zero density with (6.57), since the action in our gauge is more convenient for reading off physical quantities. Also it is readily extendable to finite baryon density.

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First we examine the KSRF relation by defining $a_{\rm KSRF}$ as

$$a_{\rm KSRF} \equiv \frac{4\, g_{v\Pi^2}^2\, f_\pi^2}{m_1^2} \sim \begin{cases} 2.03 & \text{Experiment} \\ 1.3 & \text{Sakai Sugimoto model} \end{cases}, \tag{6.58}$$

which is the same value reported in [3, 4], as expected. The universality of the vector meson coupling can be checked by $a_{\rm U}$ defined as

$$a_{\rm U} \equiv \frac{g_{v\Pi^2}}{g_{v^3}} \sim \begin{cases} 1 & \text{The universality of the vector meson coupling} \\ 0.93 & \text{Sakai Sugimoto model} \end{cases}, \tag{6.59}$$

which is also the same value as in [3]. Notice that both relations include $g_{v\Pi^2}$ and can be read from (6.57). In the $A_z = 0$ gauge we should convert $g_{v\Pi^2}$ to $a_{v\Pi^2}$ by [3]

$$a_{v\Pi^2} = \frac{2 g_{v\Pi^2}}{m_1^2}\,. \tag{6.60}$$

When we consider the field redefinition in (6.57)

$$v_\mu \to v_\mu + \frac{a_{v^3}}{2}[\Pi, \partial_\mu \Pi]\,, \tag{6.61}$$

the algebraic relation (6.60) appears immediate. However when we look at the integral expression of $g_{v\Pi^2}$ and $a_{v\Pi^2}$ the equivalence is obscured.

$$\begin{aligned} a_{v\Pi^2} &= \frac{2 g_{v\Pi^2}}{m_1^2} \\ \Leftrightarrow \frac{\pi^2}{8} \int & dZ K^{-1/3} \Psi_1 (1 - 4\widehat{\psi}_0^2) \int dZ K (\partial_Z \Psi_1)^2 = \int dZ K^{-1} \Psi_1\,. \end{aligned} \tag{6.62}$$

Next and following [3], we compare (6.57) with the action from the hidden local symmetry approach

$$\begin{aligned} S_H \equiv \mathrm{tr} \int d^4x \Big\{ & - \partial_\mu \Pi \partial^\nu \Pi + \frac{1}{2} f_{\mu\nu} f^{\mu\nu} + a g^2 f_\pi^2 v_\mu v^\nu \\ & - a g\, v_\mu [\Pi, \partial^\mu \Pi] + g\, f_{\mu\nu} [v^\mu, v^\nu] + \cdots \Big\}\,. \end{aligned} \tag{6.63}$$

The hidden local symmetry parameter (l.h.s.) can be written in terms of the D-brane effective action parameter (r.h.s.):

$$g = g_{v^3}\,, \tag{6.64}$$

$$a = \frac{2 g_{v\Pi^2}}{g} = \frac{2 g_{v\Pi^2}}{g_{v^3}}\,, \tag{6.65}$$

$$f_\pi^2 = \frac{m_1^2}{a g^2} = \frac{m_1^2}{2 g_{v^3} g_{v\Pi^2}}\,, \tag{6.66}$$

where we used the first two relations to get the last. We may define the parameter $a_{\rm H}$ which quantify the difference between hidden local symmetry approach and our model: [6]

$$a_{\rm H} \equiv \frac{2 g_{v^3} g_{v\Pi^2} f_\pi^2}{m_1^2} \sim \begin{cases} 1 & \text{Hidden local symmetry} \\ 0.72 & \text{Sakai Sugimoto model} \end{cases}, \tag{6.67}$$

[6] For a=2 the hidden local symmetry approach implies KSRF relation and the universality of the vector meson coupling. Here, we do not require this value since we want to compare our model with the hidden local symmetry itself.

which is the same value reported in [3], as expected. $a_{\rm H}$ may be interpreted as follows. Since f_π is an input parameter the Hidden local symmetry has two adjustable parameters, so a is not uniquely determined. It can be fixed by (6.65) or (6.66). When these two procedures yield the same value, $a_{\rm U} = 1$.

6.5 Numerical results

All the numerical work reported here has been carried out for the lowest modes $v_\mu \equiv B_\mu^{(1)}$ and $\Pi \equiv \varphi^{(0)}$ with the parameters discussed in section 3.

6.5.1 Mass and screening mass

From the previous section the meson masses (6.37) (time-like) and the screening masses (6.25) (space-like) are defined as

$$
\begin{aligned}
m_n &\equiv \sqrt{\lambda_n^T} M_{\rm KK}\,, \\
\mathcal{M}_n^{\shortparallel} &\equiv \sqrt{\lambda_n^\Omega} M_{\rm KK}\,, \\
\mathcal{M}_n^{\perp} &\equiv \sqrt{\lambda_n^S} M_{\rm KK}\,,
\end{aligned}
\tag{6.68}
$$

where λ_n^T, λ_n^Ω, and λ_n^S are determined as the eigenvalues of the following equations ((6.28), (6.29), (6.19)), respectively:

$$
-K^{1/3}\Delta^{-1}\partial_Z\left(K\Delta\,\partial_Z\Psi_n^T\right) = \lambda_n^T\Psi_n^T\,, \tag{6.69}
$$

$$
-K^{1/3}\Delta^{-1}\partial_Z\left(K\Delta^3\,\partial_Z\Omega_n\right) = \lambda_n^\Omega\Omega_n\,, \tag{6.70}
$$

$$
-K^{1/3}\Delta\,\partial_Z\left(K\Delta\,\partial_Z\Psi_n^S\right) = \lambda_n^S\Psi_n^S\,. \tag{6.71}
$$

Their dependense on the baryon density normalized to the nuclear matter density is shown in (3) for the lowest eigenmode. The time-like and transverse screening mass are seen to decrease midly with density. The longitudinal screening mass increases moderatly with baryon density. The mild dependence on the density for the SS model indicates that the vector mesons are weakly affected by the baryon density in this version of the SS model. As the inserted baryons are point like, at large N_c their interaction is chiefly repulsive through ω's as induced by D8-$\overline{\rm D8}$. The ω interactions with vectors and axials is mostly anomalous (through the WZ term) and therefore small as we ignored the WZ term.

6.5.2 Pion decay constant

The pion decay constant is identified from ((6.46), (6.51)) respectively,

$$
\begin{aligned}
f_\pi^{T\,2} &= \frac{4\kappa M_{\rm KK}^2}{\int dZ K^{-1}\Delta^{-3}}\,, \\
f_\pi^{S\,2} &= \frac{4\kappa M_{\rm KK}^2}{\int dZ K^{-1}\Delta^{-1}}\,.
\end{aligned}
\tag{6.72}
$$

The explicit dependence on the baryon density is shown in figure (4). Both the time-like and space-like pion decay constant are found to *increase* with the baryon density. The increase

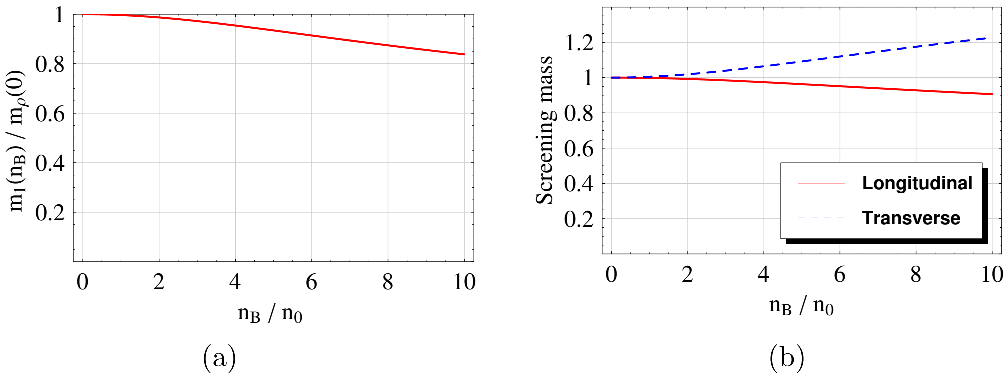


Figure 3: (a) Mass (b) Screening mass (Longitudinal mode: $\mathcal{M}_1^{\shortparallel}(n_B)/m_\rho(0)$, Transverse mode: $\mathcal{M}_1^{\perp}(n_B)/m_\rho(0)$)

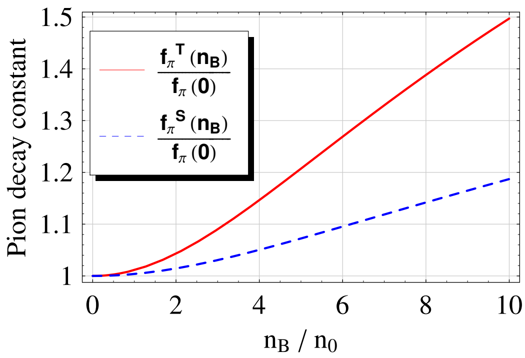


Figure 4: Pion decay constant

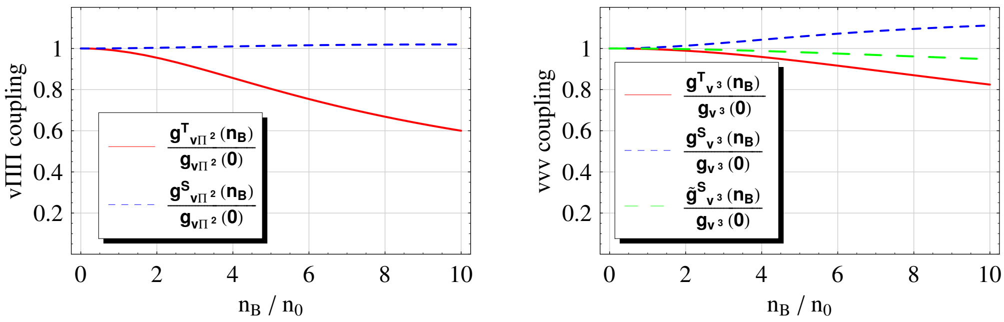
is quadratic at small densities. Since the S-wave pion scattering length with baryons is $1/N_c$ this explains the absence of a linear term. Moreover, for point-like external baryon sources the pion-Axial-Vector coupling in matter at the origin of the pion decay constant involves two baryon sources and is repulsive.

6.5.3 Vector couplings and KSRF relation

The vector couplings are identified in (6.54) and (6.56). Their overall dependence on the baryon density is again mild as explained above.

vΠΠ couplings:

$$g^T_{v\Pi^2} = \frac{1}{\sqrt{\kappa}} \frac{\int dZ \, \frac{\Omega_1}{K\Delta^3}}{\int dZ \, \frac{1}{K\Delta^3}} \, ,$$
$$g^S_{v\Pi^2} = \frac{1}{\sqrt{\kappa}} \frac{\int dZ \, \frac{\Psi_1^S}{K\Delta}}{\int dZ \, \frac{1}{K\Delta}} \, , \tag{6.73}$$



(a) (b)

Figure 5: (a) $v\Pi\Pi$ coupling (b) vvv coupling.

vvv couplings:

$$
\begin{aligned}
g^T_{v^3} &= \frac{1}{\sqrt{\kappa}} \int dZ\, K^{-1/3} \Omega_1 (\Psi^T_1)^2 \Delta \\
g^S_{v^3} &= \frac{1}{\sqrt{\kappa}} \int dZ\, K^{-1/3} (\Omega_1)^2 \Psi^S_1 \Delta^{-1} , \\
\widetilde{g}^S_{v^3} &= \frac{1}{\sqrt{\kappa}} \int dZ\, K^{-1/3} (\Psi^S_1)^3 \Delta^{-1} .
\end{aligned}
\tag{6.74}
$$

6.5.4 KSRF relations

In the matter rest frame Lorentz symmetry is no longer manifest. As a result, we expect a *variety* of KSFR relations depending on wether time-like or space-like parameters are used. Indeed, for instance the a-parameter at the origin of the KSFR relations can now take 4 different forms depending on the time-like/space-like arrangement. Specifically

$$
\begin{aligned}
a^{T1}_{\mathrm{KSRF}} &\equiv \frac{4\,(g^T_{v\Pi^2})^2\,(f^T_\pi)^2}{m_1^2} , \qquad
a^{T2}_{\mathrm{KSRF}} \equiv \frac{4\,(g^T_{v\Pi^2})^2\,(f^T_\pi)^2}{(\mathcal{M}^{\shortparallel}_1)^2} \\
a^{S1}_{\mathrm{KSRF}} &\equiv \frac{4\,(g^S_{v\Pi^2})^2\,(f^S_\pi)^2}{(\mathcal{M}^{\perp}_1)^2} , \qquad
a^{S2}_{\mathrm{KSRF}} \equiv \frac{4\,(g^S_{v\Pi^2})^2\,(f^S_\pi)^2}{(\mathcal{M}^{\shortparallel}_1)^2}
\end{aligned}
\tag{6.75}
$$

7. Conclusions

We have considered a generalization of the chiral model proposed by Sakai and Sugimoto to finite baryon density. The baryon vertices in bulk are attached equally to the D8-$\overline{\mathrm{D8}}$ branes and correspond to S^4 in $D8$. They are treated as stable and point like in $\mathbb{R}^3$ and act as uniform sources of baryon density. Their point-like nature at large N_c and coupling λ imply that their interactions as induced by D8-$\overline{\mathrm{D8}}$ is mostly repulsive through the exchanges of omega mesons.

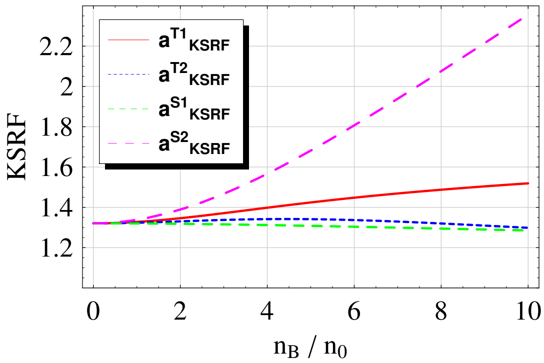


Figure 6: Generalized a-parameter

The bulk energy density grows quadratically with the baryon density before softening at asymptotic densities. The quadratic and repulsive growth is expected from the exchange of omega mesons. The softening reflects on the fact that at asymptotic densities the repulsive baryons form an instable but regular array for fixed volume V. If V acting as a container is removed, the baryons fly away in this version of the SS model. We note that the energy density scales as N_c since $N_c/\sqrt{a}$ is of order 1 as expected from standard large N_c arguments. The DBI action resums (partially) the strong NN-interactions while keeping the leading N_c result unchanged. Since the instanton size is of order $1/\sqrt{\lambda}$ we also note that the resummed contributions are of order λ^0 since the bulk instanton density $\sqrt{\lambda}n_B$ is of order λ^2 (The additional $\sqrt{\lambda}$ here stems from the rescaling of $z \to z/\sqrt{\lambda}$ in the delta-function source at $z = 0$).

Using linear response theory, we have probed this dense baryonic system using pions, vectors and axials. The point like nature of the baryons with a size of order $1/\sqrt{\lambda}$ and the large N_c nature as noted above, causes rather mild changes in the masses and couplings as a function of baryon density. In contrast, the pion decay constants are found to change appreciably. The quadratic increases at small baryon densities is mediated by omega's. The scalar S-wave pion-baryon scattering length is noted to vanish at large N_c, causing f_π to increase instead of decreasing at finite density. This behaviour is unphysical.

The current approach needs to be improved in a number of ways to accomodate the baryon physics expected in the real world. First, the point-like nature of the sources need to be relaxed. This is possible by constructing the pertinent instanton vertex. Also, the point-like limit suggests that the DBI results quoted here are only indicative since higher derivative corrections to the DBI effective action are expected to contribute (see also [3 − 5] for further comments on this point)). Second, the Fermi motion of the sources need to be included. This can be achieved through a select quantization of the collective variables associated to the baryon vertex insertion. Some of these issues will be addressed in later work.

Note added. After the completion of this work, we became aware of the recent work by O. Bergman, G. Lifschytz, and M. Lippert [15] who also address the SS model at finite baryon density. They have shown that a cusp configuration develops at finite density for generically separated D8-$\overline{\mathrm{D8}}$. This observation does not apply to the original SS embedding we discuss here. We also noticed the appearance of two relevant papers: [16] discusses the finite density problem in the holographic NJL model, and [17] discusses the effects of a finite size baryon charge distribution.

Acknowledgments

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A. The effective action using the vacuum modes

Let us only consider the lightest vacuum meson modes corresponding to the pion and ρ meson fields. This vacuum mode decomposition was studied in [3] at zero baryon density. Here we just add $\mathcal{A}_0$ as obtained in section 3 to the gauge field A_M. Since the mode decomposition is complete, this approach should be complementary to the one discussed in the text. It is the same as the one we used in [8]. As we will show, the results are overall similar to the ones discussed in the main text regarding the density dependence.

In the gauge $A_z = 0$ and $\xi \equiv e^{\frac{i\Pi(x^\mu)}{f_\pi}}$ (5.24), A_μ reads[7]

$$\begin{aligned} A_\mu(x^\mu, z) &= -i\mathcal{A}_0(z) + v_\mu(x^\mu)\,\psi_1(z)\ . \\ &\quad + \left(\frac{2i}{f_\pi}\partial_\mu\Pi + [\partial_\mu\Pi^3]\right)\widehat{\psi}_0(z) + \frac{1}{2f_\pi^2}[\Pi, \partial_\mu\Pi] + \mathcal{O}(\Pi^4)\,, \end{aligned} \tag{A.1}$$

where $\mathcal{A}_0$ is the background field, $v_\mu \equiv B_\mu^{(1)}$. We have set $B_\mu^{(n)} = 0$ for $n \geq 2$. The corresponding field strengths are

$$\begin{aligned} F_{\mu\nu} &= (\partial_\mu v_\nu - \partial_\nu v_\mu)\psi_1 + [v_\mu, v_\nu]\psi_i^2 \\ &\quad + \frac{2i}{f_\pi}([\partial_\mu\Pi, v_\nu] + [v_\nu, \partial_\nu\Pi])\psi_1\widehat{\psi}_0 + \frac{1}{f_\pi^2}[\partial_\mu\Pi, \partial_\nu\Pi](1 - 4\widehat{\psi}_0^2) + \mathcal{O}((\Pi, v_\mu)^3) \end{aligned} \tag{A.2}$$

$$F_{z\mu} = -i\dot{\mathcal{A}}_0 + \left(\frac{2i}{f_\pi}\partial_\mu\Pi + [[\partial_\mu\Pi^3]]\right)\widehat{\phi}_0 + v_\mu\dot{\psi}_1 + \mathcal{O}(\Pi^4) \tag{A.3}$$

where $\dot{\mathcal{A}}_0 = \frac{d\mathcal{A}_0}{dz}$, $\dot{\psi}_1 = \frac{d\psi_1}{dz}$, and

$$\widehat{\phi}_0 = \partial_z\widehat{\psi}_0 = \frac{1}{\pi U_{\mathrm{KK}}}\frac{1}{K} \sim \phi_0 \text{ in (5.7)} \tag{A.4}$$

[7] In this section the gauge field A_μ is treated as anti-Hermitian. $\mathcal{A}_0$ and Π is Hermitian so i was introduced, while v_μ is anti-Hermitian. Note that we are working in a different gauge from section 6.

$F_{\mu\nu}F^{\mu\nu}$	$\rightarrow$	$f_{\mu\nu}f^{\mu\nu}\psi_1^2$	$\equiv$	α_2
		$2f_{\mu\nu}[v^\mu, v^\nu]\psi_1^3 + \frac{2}{f_\pi^2}f_{\mu\nu}[\partial^\mu\Pi, \partial^\nu\Pi]\psi_1(1-4\widehat{\psi}_0^2)$	$\equiv$	α_3
$F_{\mu z}F^{\mu z}$	$\rightarrow$	$(\dot{\mathcal{A}}_0)^2$	$\equiv$	β_0
		$-\frac{4}{f_\pi}(\partial_0\Pi)\widehat{\phi}_0\dot{\mathcal{A}}_0 + 2iv_0\dot{\psi}_1\dot{\mathcal{A}}_0$	$\equiv$	β_1
		$-\frac{4}{f_\pi^2}(\partial_\mu\Pi\partial^\mu\Pi)\widehat{\phi}_0^2 + v_\mu v^\mu\dot{\psi}_1^2 + \frac{2i}{f_\pi}\{\partial_\mu\Pi, v^\mu\}\widehat{\phi}_0\dot{\psi}_1$	$\equiv$	β_2
$[F^4]$	$\rightarrow$	$f_{ij}f^{ij}(\dot{\mathcal{A}}_0)^2\psi_1^2$	$\equiv$	γ_2
		$\Big[2f_{ij}[v^i, v^j](\dot{\mathcal{A}}_0)^2\psi_1^3 + \frac{2}{f_\pi^2}f_{ij}[\partial^i\Pi, \partial^j\Pi](\dot{\mathcal{A}}_0)^2\psi_1(1-4\widehat{\psi}_0^2)$		
		$-2iv_0f_{ij}f^{ij}\dot{\mathcal{A}}_0\dot{\psi}_1\psi_1^2\Big]$	$\equiv$	γ_3

Table 3: The relevant terms in evaluating DBI action up to third order in the fields (Π, v). All entries are understood in the integral and trace operation.

Notice that $\mathcal{A}_0$ does not contribute to $F_{\mu\nu}$ and affect only $F_{z\mu}$.

In order to compute the DBI action (5.9),

$$S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}} = -\widetilde{T}\int d^4x dz\, U^2$$
$$\mathrm{tr}\,\sqrt{1-(2\pi\alpha')^2\frac{R^3}{2U^3}F_{\mu\nu}F^{\mu\nu} - (2\pi\alpha')^2\frac{9}{4}\frac{U}{U_{\mathrm{KK}}}F_{\mu z}F^{\mu z} + [F^3] + [F^4] + [F^5]}\,,$$

we need to know $F_{\mu\nu}F^{\mu\nu}, F_{\mu z}F^{\mu z}, [F^3], [F^4]$, and $[F^5]$, which have many complicated contributions. Again, we use the observations noted in the text to simplify. Thus

$$[F^4] = (2\pi\alpha')^4\frac{9}{8}\frac{U}{U_{\mathrm{KK}}}\left(\frac{R}{U}\right)^3 F_{0z}F^{0z}F_{ij}F^{ij} + \mathcal{O}((v_\mu, \varphi)^4)\ . \tag{A.5}$$

Table (3) lists all relevant terms. We have introduced $f_{\mu\nu}$ defined as

$$f_{\mu\nu} \equiv \partial_\mu v_\nu - \partial_\nu v_\mu\,, \tag{A.6}$$

with $\mu\nu = 0, 1, 2, 3$ and $i, j = 1, 2, 3$. Table (3) should be understood in the integral and trace operation. We have omitted some terms vanishing in the operation and rearranged some terms by using the cyclicity of the trace.

In terms of the entries in the r.h.s. of the table, the action reads

$$S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}} = -\widetilde{T}\int d^4x dz\, U^2\,\mathrm{tr}\,\sqrt{P_0 + P_1}\,, \tag{A.7}$$

with

$$P_0 \equiv 1 - (2\pi\alpha')^2\frac{9}{4}\frac{U}{U_{\mathrm{KK}}}\ \beta_0 = 1 - bK^{\frac{1}{3}}(\partial_Z\mathcal{A}_0)^2\,, \tag{A.8}$$

$$P_1 \equiv (2\pi\alpha')^2\frac{R^3}{2U^3}(\alpha_2+\alpha_3) + (2\pi\alpha')^2\frac{9}{4}\frac{U}{U_{\mathrm{KK}}}(\beta_1+\beta_2) + (2\pi\alpha')^4\frac{9}{8}\frac{R^3}{U_{\mathrm{KK}}U^2}(\gamma_2+\gamma_3)\ . \tag{A.9}$$

Again, P_0 does not include meson fields and has carries the baryon density. Expanding the action by fluctuating the fields we have

$$
\begin{aligned}
S^{\mathrm{DBI}}_{\mathrm{D8}\text{-}\overline{\mathrm{D8}}} &= -\widetilde{T}\int d^4x dz\, U^2\, \mathrm{tr}\, \left[\sqrt{P_0}+\frac{1}{2}\frac{P_1}{\sqrt{P_0}}-\frac{1}{8}\frac{P_1^2}{\sqrt{P_0}^3}+\frac{1}{16}\frac{P_1^3}{\sqrt{P_0}^5}\right]+\mathcal{O}((\Pi,v_\mu)^4) \\
&= S_1+S_2+\mathcal{O}((\Pi,v_\mu)^4)\,,
\end{aligned}
\tag{A.10}
$$

with

$$
S_1 \equiv -\widetilde{T}\int d^4x dz\, U^2 \mathrm{tr}\ \Delta^{-1}\,, \tag{A.11}
$$

$$
S_2 \equiv -\widetilde{T}\int d^4x dz\, U^2 \mathrm{tr}\ \left[\frac{1}{2}\Delta P_1-\frac{1}{8}\Delta^3 P_1^2+\frac{1}{16}\Delta^5 P_1^3\right]\,, \tag{A.12}
$$

where we defined a modification factor $\Delta(Q)$ as

$$
\Delta(Q) \equiv \frac{1}{\sqrt{P_0}} = \frac{1}{\sqrt{1-bK^{\frac{1}{3}}(\partial_Z\mathcal{A}_0)^2}} = \sqrt{1+\frac{n_B^2}{4a^2b}K^{-5/3}}\,.
$$

Notice that $-S_1$ is the grand potential discussed in section 3, and S_2 will be reduced to the action of mesons. To accomplish it we plug (A.9) into (A.12) and evaluate all z integrals and identify them as coefficients of each term in the remaining 4-D action.

Let us first check which terms we have and how they are affected by finite baryon density schematically. It can be read off from table (3). At zero density we set $\mathcal{A}_0 = 0$ and $\Delta = 1$. Then $\alpha_2, \alpha_3, \beta_2$ survive. α_2, β_2 correspond to the free action of Π and ρ, and α_3 is the couplings of vvv, $v\Pi\Pi$ interaction. At finite density all terms are enhanced by Δ, Δ^2,or Δ^3. Furthermore there are nontrivial modification. The free action part will be affected by γ_2 and β_1^2. (β_1 itself does not contribute because the first term has odd parity in z and the second term is traceless.) The couplings are modified by γ_3. There are new interaction terms such as $v_0(\partial_\mu v_\nu - \partial_\nu v_\mu)^2$, $v_0 v_\mu v^\mu$, $v_0\partial_\mu\Pi\partial^\mu\Pi$, $\partial_0\Pi\{\partial_\mu\Pi, v^\mu\}$, which all vanish at zero density.

Considering all these modification we get the final form of the meson action

$$
\begin{aligned}
S_2 = \int d^4x \Big[& -a^T_{\Pi^2}\mathrm{tr}\ \left(\partial_0\Pi\partial^0\Pi\right) - a^S_{\Pi^2}\mathrm{tr}\ \left(\partial_i\Pi\partial^i\Pi\right) \\
& + a^T_{v^2}\mathrm{tr}\, f_{0i}f^{0i} + \frac{1}{2}a^S_{v^2}\mathrm{tr}\, f_{ij}f^{ij} + m_v^{2\,T}\,\mathrm{tr}\, v_0 v^0 + m_v^{2\,S}\,\mathrm{tr}\, v_i v^i \\
& + a^T_{v^3}\mathrm{tr}\left(2f_{0i}[v^0,v^i]\right) + a^S_{v^3}\mathrm{tr}\left(f_{ij}[v^i,v^j]\right) \\
& + a^T_{v\Pi^2}\mathrm{tr}\left(2f_{0i}[\partial^0\Pi,\partial^i\Pi]\right) + a^S_{v\Pi^2}\mathrm{tr}\left(f_{ij}[\partial^i\Pi,\partial^j\Pi]\right) + \cdots \Big]\,,
\end{aligned}
\tag{A.13}
$$

where the coefficients of every term are defined in table (4). At zero density all coefficients agree with those in [3]. There are three types of modification due to the baryon density: Δ, Δ^{-1}, and Δ^3. Δ simply comes from $\frac{1}{2}\Delta P_1$ in (A.12). Since Δ is common to all coefficients, Δ^3 and Δ^{-1} can be understood as $\Delta\cdot\Delta^2$ and $\Delta\cdot\Delta^{-2}$. An enhancing factor Δ^2 is

Coefficients	Definition	$Q = 0(\Delta = 1)$
$a^T_{\Pi^2}$	$\frac{1}{\pi}\int dZ K^{-1}\,\Delta^3$	1
$a^S_{\Pi^2}$	$\frac{1}{\pi}\int dZ K^{-1}\,\Delta$	1
$a^T_{v^2}$	$\int dZ\, K^{-\frac{1}{3}}\Psi_1^2\,\Delta$	1
$a^S_{v^2}$	$\int dZ\, K^{-\frac{1}{3}}\Psi_1^2\,\Delta^{-1}$	1
m_v^{2T}	$M^2_{\rm KK}\int dZ\, K(\partial_Z\Psi_1)^2\,\Delta^3$	m^2_ρ
m_v^{2S}	$M^2_{\rm KK}\int dZ\, K(\partial_Z\Psi_1)^2\,\Delta$	m^2_ρ
$a^T_{v^3}$	$\frac{1}{\sqrt{\kappa}}\int dZ\, K^{-\frac{1}{3}}\Psi_1^3\,\Delta$	$\frac{1}{\sqrt{\kappa}}\cdot 0.446$
$a^S_{v^3}$	$\frac{1}{\sqrt{\kappa}}\int dZ\, K^{-\frac{1}{3}}\Psi_1^3\,\Delta^{-1}$	$\frac{1}{\sqrt{\kappa}}\cdot 0.446$
$a^T_{v\Pi^2}$	$\frac{1}{\sqrt{\kappa}}\frac{\pi}{4M^2_{\rm KK}}\int dZ\, K^{-1/3}\Psi_1(1-4\widehat{\psi}_0^2)\Delta$	$\frac{1}{\sqrt{\kappa}}\frac{\pi}{4M^2_{\rm KK}}\cdot 1.584$
$a^S_{v\Pi^2}$	$\frac{1}{\sqrt{\kappa}}\frac{\pi}{4M^2_{\rm KK}}\int dZ\, K^{-1/3}\Psi_1(1-4\widehat{\psi}_0^2)\Delta^{-1}$	$\frac{1}{\sqrt{\kappa}}\frac{\pi}{4M^2_{\rm KK}}\cdot 1.584$

Table 4: The definitions of the coefficients in the action (A.13). At finite density, there are enhancing factors Δ, Δ^3 and a suppressing factor Δ^{-1}.

Coefficients	Definition	$Q = 0(\Delta = 1)$
$g^T_{v\Pi^2}$	$\sqrt{\kappa}M^2_{\rm KK}U^2_{\rm KK}\int dZ\, K\Delta^3\Psi_1\phi_0^2$	$\frac{1}{\sqrt{\kappa\pi}}\cdot 0.63$
$g^S_{v\Pi^2}$	$\sqrt{\kappa}M^2_{\rm KK}U^2_{\rm KK}\int dZ\, K\Delta\Psi_1\phi_0^2$	$\frac{1}{\sqrt{\kappa\pi}}\cdot 0.63$

Table 5: The definitions of the coefficients in the action (A.13). At finite density, there are enhancing factors Δ, Δ^3 and a suppressing factor Δ^{-1}.

due to additional contribution from β_1^2 and a suppressing factor Δ^{-1} is from γ_2, γ_3, which explains the calculational similarities in all the results.

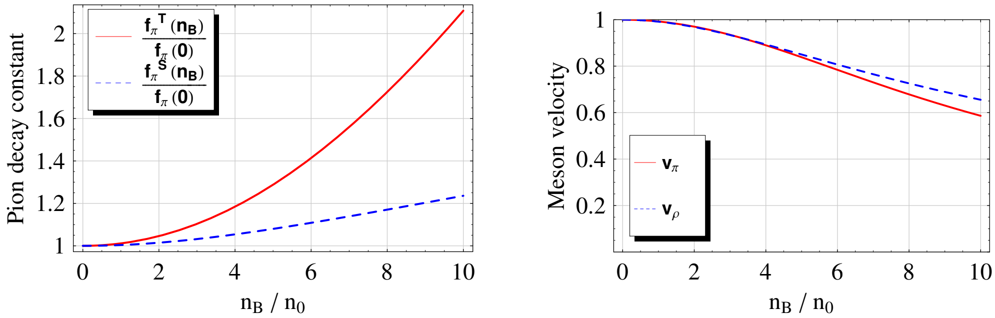
Finally we want to mention without details, the character of the action in the gauge $A_M(z \to \infty) \to 0$ instead of $A_z = 0$. The action is the same as in (A.13) except for the interaction term $v\Pi\Pi$

$$-2g^T_{v\Pi^2}\, v_0[\Pi, \partial^0\Pi] - 2g^S_{v\Pi^2}\, v_i[\Pi, \partial^i\Pi]\,, \tag{A.14}$$

where $g^T_{v\Pi^2}$ and $g^S_{v\Pi^2}$ are defined in table(5).

A.1 Numerical results

In this section we compute the coefficients in table (4), (5) numerically. Their physical



(a) (b)

Figure 7: (a) Pion decay constant vs $\frac{n_B}{n_0}\left[\frac{f_\pi^T}{f_\pi} = \sqrt{a_{\pi^2}^T}, \frac{f_\pi^S}{f_\pi} = \sqrt{a_{\pi^2}^S}\right]$, (b) Velocity of Π and ρ (v) vs $\frac{n_B}{n_0}\left[v_\pi \equiv \sqrt{\frac{a_{\pi^2}^S}{a_{\pi^2}^T}} \text{ and } v_v \equiv \sqrt{\frac{a_{v^2}^S}{a_{v^2}^T}}\right]$

meanings can be read off from the action (A.13). We use the same numerical inputs as detailed in section 3.

A.1.1 Pion decay constant

The pion decay constant can be defined by the procedure of section 6.3 with the vacuum mode function.

$$f_\pi^T \equiv f_\pi \sqrt{a_{\pi^2}^T}\,, \qquad f_\pi^S \equiv f_\pi \sqrt{a_{\pi^2}^S} \tag{A.15}$$

A.1.2 Velocity

The pion velocity:

$$v_\pi \equiv \sqrt{\frac{a_{\pi^2}^S}{a_{\pi^2}^T}} = \frac{f_\pi^S}{f_\pi^T} \tag{A.16}$$

The lowest mode velocity:

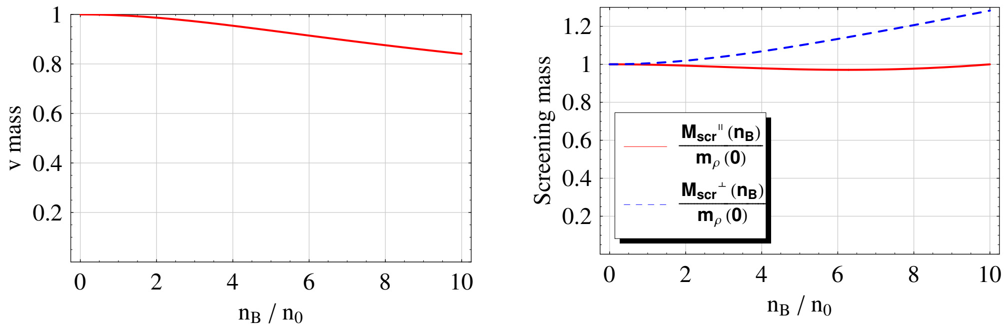
$$v_v \equiv \sqrt{\frac{a_{v^2}^S}{a_{v^2}^T}} \tag{A.17}$$

A.1.3 Mass

$$M_1 \equiv \sqrt{\frac{m_v^{2S}}{a_{v^2}^T}} \tag{A.18}$$

A.1.4 Screening mass

$$M_{\rm scr}^{\shortparallel} \equiv \sqrt{\frac{m_v^{2T}}{a_{v^2}^T}}\,, \qquad M_{\rm scr}^{\perp} \equiv \sqrt{\frac{m_v^{2S}}{a_{v^2}^S}}\,. \tag{A.19}$$



(a) (b)

Figure 8: (a) v mass vs $\frac{n_B}{n_0}$ $\left[\frac{M_1}{m_\rho} \equiv \sqrt{\frac{m_v^{2S}}{a^T_{v^2} m_\rho^2}}\right]$, (b) Screening masses vs $\frac{n_B}{n_0}$ $\left[\frac{M^{\shortparallel}_{\rm scr}}{m_\rho} \equiv \sqrt{\frac{m_v^{2T}}{a^T_{v^2} m_\rho^2}}\right.$, $\left.\frac{M^{\perp}_{\rm scr}}{m_\rho} \equiv \sqrt{\frac{m_v^{2S}}{a^S_{v^2} m_\rho^2}}\right]$

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